International Journal of Recent Advances in Multidisciplinary Topics
Volume 2, Issue 10, October 2021
https://www.ijramt.com | ISSN (Online): 2582-7839

On finding the patterns and Inter-Dependence of Key Factors in the Economic Growth of India

Mudit Rastogi*
University of Pune, Pune, India

***Abstract*: In the past and recent years a number of methods have been devised to find the critical factors affecting the economic growth of one or a group of countries. These methods are mostly statistics based, using correlation techniques to evaluate the inter dependence and the trends in an economy. In this article, I present a new graphical method; Recurrence Plot (RP) and Recurrence Quantitative Analysis (RQA) technique, to visualize the patterns and compute the interdependence of key factors that affect the economic growth of a country. To demonstrate this technique, I choose India, as a country example. The reason mainly being twofold, firstly it's one of the fastest growing economy in the world, and as a result the conclusions and trends found from this article may be useful for other studies, mainly in the field of public policy, social studies, governance etc. Secondly, the author has a better understanding of Indian economy and its market dynamics.**

***Keywords*: Recurrence Plot, Recurrence Quantitative Analysis, Economic Growth, Pattern, Key factors**

1. Introduction

Since the early 20th century numerous economists using both the theory and empirical research, have tried to show and find the causes of economic growth. In the beginning, growth theory was consisted mainly of the "neoclassical model", as developed by Ramsey (1928), Solow (1956), Swan (1956), Cass (1965), and Koopmans (1965). Ramsay conceptualized a sophisticated model of society's optimal saving i.e. the allocation of resources chosen optimally by a planner that tries to maximize the utility of household. But this model was mathematically very demanding. The model was later studied by David Cass and Tjalling Koopmans in 1965 to extend it to a decentralized environment where households supply labor, hold capital and consume optimally, for a given prices and wages, while firms rent capital and hire labor to maximize the profits. This model is often referred to as the Ramsey-Cass-Koopmans model. Later Ramsey model was fusioned with Solow's simpler growth model (Solow 1956) which became a cornerstone in neoclassical growth theory since the mid 1960's. Solow model explained the long-run economic growth by looking at capital accumulation, labor or population growth, and increase in productivity, commonly grouped to as technological progress. Its main conclusion that the long term growth in standards of living across countries cannot be accounted for, simply by the accumulation of physical capital was a key message.

As the time progressed, these neo-classical models gave rise to "endogenous growth" theories, as the former predicted economies without technological changes and thus all of them would eventually converge to a steady state, with zero per capita growth. In the endogenous growth theory it was put forward, that the long-term growth rate was determined by government policies and other forces contained in the analysis. Some of the main work in this field, was done by Romer and Rebelo amongst others. Romer (1990) asserted that the raw materials that we use have not changed much throughout time, but as a result of technical progress the instructions that we follow to combine them have become vastly more sophisticated. Thus, technical progress is the driving force behind economic growth and so should be modelled endogenously. Rebelo (1991) presented a model commonly known as "Rebelo AK Growth Model" in which a social planner maximizes the discounted sum of utility in an economy with a calculated AK production function. In fact, the Rebelo AK model is essentially just a way of reinterpreting the perfect foresight infinite horizon consumption problem as a model for economic growth. The key distinction is that we generally use the perfect foresight infinite horizon model to analyze circumstances where the agent has both labor and capital income, whereas the Rebelo model rules out labor income by assumption.

Following their work, Robert Barro (1996) conducted extensive empirical work to test the established hypotheses. He analyzed a total of 100 countries from 1960 to 1990 and came out with crucial factors that affect the economic growth of countries. He found out in his research that the growth rate of real per capita GDP is associated with maintenance of the rule of law, smaller government consumption, longer life expectancy, more male secondary and higher levels of schooling, lower fertility rates, higher levels of investment, the level of democracy, a lower inflation rate, and openness to trade. He also emphasized the theory of convergence, which implies that as the real GDP level rises, the growth rate falls. In view of this, and taking lead from the Barro, Romero, Solow & Swan models, I shortlist some of the key factors which have the strongest impact on economic growth of a country. These are discussed in the Data & Methodology section. I then investigate

*Corresponding author: mudit.r@hotmail.com

the possibility to use Recurrence Plot (RP) and Recurrence Quantification Analysis (RQA) technique to detect the patterns and the effect of these key factors on the overall growth and development of India. One important question which arises now, is whether an economy growth time series and its equivalent should be studied from the point of view of stochastic process, or non-linear simple deterministic systems which can exhibit the phenomenon of deterministic chaos. The answer to this question is an open debate, and will not be discussed in detail in this article. In this article, I model the economic growth time series as a non-linear dynamical system. However it should be kept in mind that, RP and RQA techniques here are not intended to prove any evidence of chaos, but are used for their sheer credibility in detecting changes in data behavior, in particular phase transitions and other dynamic properties of a time series. Though most of the RP and RQA applications are to be seen in the field of medicine, biology and biomedical, some researchers have also applied it to the financial data (McKenzie 2001, Storzzi 2002).

The presented article is an analysis study to identify rare events, patterns, sudden discrepancies in the form of surge or sulk using RP and RQA techniques. The analysis is done on the Indian Economic data, taken over a time span of 49 years from 1967 to 2015. The paper is divided into two parts. The first part talks about the RP and RQA theory. The second part is dedicated to data management, results, interpretation and analysis. In the end, on the basis of interpretation and results, some conclusions are drawn which would help identifying different socio-economic factors affecting the Indian economy, and which must be considered seriously when making plans for business growth.

2. Technical Overview

1) Concept of Phase Space

Phase space can be defined as a space in which all the possible states of a system are represented with each corresponding to different points in the phase space. It is true that the possibility of a detailed analysis of any dynamic system can be accomplished if all the changing states of the system are represented by state vectors in the phase space. However, it is realizable only if all the degrees of freedom and equations of motion are known, but in reality we can observe a very few of these quantities in the system. Nevertheless, it is in fact possible to reconstruct the entire dynamics of a system from this relatively small number of observables, as all the different degrees of freedom of a dynamic system always interact with each other. Therefore it can be said that the combination of all other components is concealed in each observable quantity through the main state vector components. According to Taken (1981) it is possible to reconstruct a phase space trajectory from a single scalar time series of observable quantities such as (x1, …. , xn). On doing so, it is possible to analyze the system by examining the dynamics in a space defined by delayed vectors of dimension "m". Here m represents the dimensional space used to construct the unknown points of the phase space at any time i by reconstructing the delayed vector y(i). The delayed vector y(i), can be built as y(i) = xi, xi+d, ...,xi+(m−1)d, where "d" and "m" are called respectively the time delay and the embedding dimension. This phase space is also called the "reconstructed phase space". I would discuss in detail on how to find the correct embedding parameter values in the Methodology section later in this article.

2) Recurrence Plot

The concept of recurrence goes back to Poincare (1890), who proved that after a sufficiently long time, the trajectory of a chaotic system in phase space will return close to any of the former point in its route with a probability one. Later on, Recurrence Plots (RPs) were introduced by Eckmann (1987) to visualize the behavior of trajectories of dynamical systems in a phase space. Suppose that the time series $x_i\}^{N}{}_{i=1}$ represents the trajectory of a system in phase space is given, with $x_i \epsilon R^d$. The RP is then based on the following matrix

$$Ri,j = \Theta\left(\varepsilon - \left||xi - xj|\right|\right), \qquad i,j = 1,\ldots,N$$

Where Θ (.) is the heavy side function, ||.|| denotes a norm and Ɛ is the predefined threshold value. I will use the maximum norm and a threshold value of 0.1 in this article. After using the above equation, the value 1 is coded as a black dot and the value zero as white one in the RP. As a result, I get a 2-dimensional N x N matrix, which is symmetric to the main diagonal i = j. technically speaking, the matrix elements corresponds to those times at which a state of a dynamical system visits roughly the same area in the phase space. The black dots represent the rare events, surge or sulk, times at which the data was inconsistent. On closer inspection of the RPs many small structures would be revealed which are single dots, diagonal, vertical and horizontal lines. All of them speaks a lot about the time series. Single, isolated recurrence points occur when the states are rare. A diagonal line is seen when a segment of the trajectory runs parallel to another segment, i.e. the trajectory visits the same region of the phase space at different times. The length of this diagonal line tells the duration of such similar local evolution of the trajectory segments. A vertical line marks a time length in which a state does not change or changes very slowly. I have summarized all these observations in the Table 1 below. A

Table 1
Recurrence Plots and its Interpretation

Observation	Interpretation
Homogeneity	Stationary Process
Fading to the upper left and lower right corners	Non stationary process containing a trend or a drift
Disruptions (white bands) occur	Nonstationary; some states are rare or far from the normal; transitions may have occurred
Periodic/ quasi-periodic patterns	Cyclicities in the process
Single isolated points	Heavy fluctuation in the process
Diagonal lines (parallel to the main diagonal)	The evolution of states is similar at different times; the process could be deterministic
Diagonal lines (orthogonal to the main diagonal)	The evolution of states is similar at different times but with reverse time
Vertical and horizontal lines/clusters	Some states do not change or change slowly for some time; indication for laminar states
Long bowed line structures	The dynamics of the system could be changing

detailed description on the subject of RPs is given in Marwan (2007).

3) Recurrence Quantitative Analysis

After the success of RPs, in due time the concept of Recurrence Quantitative Analysis (RQA) was introduced by Webber (1992). Later Marwan (2002) and Trulla (1996) performed research work in this area and came out with new measures. The primary motivation was to quantify the images found in RPs because the detection of subtle patterns in the RPs using qualitative visual inspection is difficult and subjective. The measures used in this study are based on detection and measurement of distribution of diagonal and vertical structures and are interpreted as following;

- *Recurrence Rate RR*: It is defined as the percentage of black points in the RP. It signifies the ratio of the number of recurrent states measured with respect to all possible states.

$$RR = \frac{1}{N^2}\sum_{i,j=1}^{N} Ri,j = \Theta\left(\varepsilon - ||xi - xj||\right)$$

- *Determinism:* It is defined as the percentage of black points which are part of diagonal lines of length l_{min}.

$$DET = \frac{\sum_{l=lmin}^{N} l\,P(l)}{\sum_{l=1}^{N} l\,P(l)}$$

Where P (l) denotes the probability of finding a diagonal line of length l in the RP. This talks about how predictable a system is. For a periodic system, one gets DET = 1 and for a purely stochastic system DET tends to zero.

- *Divergence:* It is defined as

$$DIV = \frac{1}{Lmax}$$

Where L_{max} is the length of the longest diagonal other than the main diagonal. It is also a measure of the largest Lyapunov exponent.

- *Ratio:* It is simply the ratio between DET and RR,

$$RATIO = \frac{DET}{RR}$$

- *Laminarity:* It is the percentage of recurrence points that form the vertical lines, v in the RP

$$LAM = \frac{\sum_{v=vmin}^{N} v\,P(v)}{\sum_{v=1}^{N} v\,P(v)}$$

- *Entropy*: It is the Shannon entropy of the frequent distribution of diagonal lines in the plot.

$$ENTR = -\sum_{l=lmin}^{N} p(l)\ln p(l)$$

- *Trend:* It is a linear regression coefficient over the recurrence rate on each diagonal line parallel to the main diagonal.

$$TREND = \frac{\sum_{i,=1}^{N}(i - \frac{N}{2})(RRi - < RRi >)}{\sum_{i=1}^{N}(i-N)^2}$$

- *Trapping Time:* The average length of the diagonal line

$$TT = \frac{\sum_{v=vmin}^{N} vP(v)}{\sum_{v=vmin}^{N} P(v)}$$

4) Cross Recurrence Plots (CRP)

CRP is an extension of RP and was introduced to analyze the dependence between two different systems. They are in a way generalization of the linear cross correlation function. For example if we have two systems, each being represented by the trajectories $\{x_i\}$ and $\{y_i\}$ in a d- dimensional phase space for i = 1, ….. , N, then the CRP can be defined as

$$CR\,i,j = \Theta\left(\varepsilon - ||xi - yj||\right), \qquad i,j = 1,\ldots,N$$

The notation is analogous to the definition of RPs. If in the second trajectory of a state at time j is close to a state on the first trajectory at time i, a black dot will be assigned to the matrix CR at location (i,j). This occurrence of neighbors in both trajectories is not a recurrence of states and therefore the matrix represents the conjunctures of states of both the systems. It should be noted that both the trajectories for the creation of a CRP have to represent the same dynamical system with equal state variables because they are in the same phase space. This must be taken into account if time series of different measurements are involved. If in case, both the trajectories come from the same process but have different absolute values, the CRP will not yield the expected RP if a fixed threshold Ɛ is chosen. Therefore, it is necessary to adapt both trajectories to the same range of values by using a normalization to the standard deviation. Since the values of $CR_{i,i}$ (i = 1,……., N) are not necessarily one, the black main diagonal usually vanishes (Figure 12). However the lines which are more or less diagonally oriented are of interest. The frequency and lengths of these lines are obviously related to a certain similarity between the dynamics of both sub-systems.

5) Joint Recurrence Plots (JRP)

The idea of the joint recurrence is based on the concept of mutual information, which quantifies the amount of information that we obtain from the measurement of one variable on another. As we have seen in the previous section, CRPs cannot be used for the analysis of two physically different time series, because the difference between two vectors with different physical units or even different phase space dimension does not make sense. As a result, another possibility to compare different systems is to consider the recurrence of their trajectories in their respective phase spaces simultaneously. Joint Recurrence does exactly the same by telling us the probability that both systems recur simultaneously to the neighborhood of a formerly visited point in their respective phase space. In order to estimate the mutual information I consider two matrices $R^{x}_{i,j}$ and $R^{y}_{i,j}$ separately. I do not mix the phase spaces of x and y, as in CRP. I rather extend the phase space to R^{d1+d2}, where d1 and d2 are the phase space dimensions of the corresponding subsystems, which are in general different. Additionally, I consider two different thresholds for each system Ɛx and Ɛy respecting the natural measure of both systems. The Joint Recurrence matrix is thus denoted by as:

$$JR^{xy}\,i,j = \Theta\left(\varepsilon^{x} - ||xi - xj||\right)\Theta\left(\varepsilon^{y} - ||yi - yj||\right), \qquad i,j = 1,\ldots,N$$

The JR plot is defined as

$$JR^{xy}\ i,j = \begin{cases} 1, & if\ ||xi - xj|| < \varepsilon^{x}\ and\ ||yi - yj|| < \varepsilon^{y} \\ 0, & else \end{cases}$$

In this method a recurrence takes place if one point of the trajectory x_j for j = 1, 2… returns to the close neighborhood of a former point x_i in phase space, and simultaneously another point of the trajectory y_j for j = 1, 2… returns to the close neighborhood of a former point y_i. In other words, I calculate the joint probability that both recurrences happen simultaneously in their respective sub-phase spaces. To conclude, it is reasonable to say that CRPs are more appropriate to investigate relationships between the parts of the same system which have been subjected to different physical or mechanical processes. On the other hand, the JRPs are more appropriate for the investigation of two interacting systems which influence each other, and hence adapt to each other, e.g. in the framework of phase and generalized synchronization.

3. Data and Methodology

On the basis of the literature review of "neo classical" and "endogenous growth" theory models as discussed in the Introduction part, I selected the critical factors I hope to have an influence on the economic growth of a country. The **Error! Reference source not found.** below describes each of these key factors, in detail with their predicted (assumed) effects on the economy based on the literature review. All the data was taken from the World Bank- data bank spanning from 1967 to 2015, a total of 49 years. For few factors, the data is unavailable during the given time period, and hence the analysis was adjusted accordingly. I didn't interpolate or extrapolate. This is because I am more interested in seeing the trends and patterns rather than a complete statistical analysis.

Table 2
Key Economic Factors and their Impact

Key Factors	Description (Social or Economic Factor)	Unit	Predicted effect
Exports of goods and services	Economic	Current USD	+
Central Government Debt	Economic	% of GDP	-
Gross Fixed Capital Formation	Economic	Current USD	+
Foreign Direct Investments	Economic	% of GDP	+
Inflation, consumer prices	Economic	Annual %	-
Labor Force	Social	Total headcounts	+
Research & Development Expenditure	Economic	% of GDP	+
GINI Index	Social	Index Value	+/-
Official aid received	Economic	Current USD	+/-

1) Exports

The relationship between exports and economic growth has been a popular subject of debate and discussion among the economists, since the late 1970s. Early studies relied on correlation coefficients between export growth and economic growth as in Michaely (1977) Michalopoulos (1973). In the 1980s, most studies used the Granger causality test method to investigate lead-and-lag relations. Notable examples include Chow (1987), Jung and Marshall (1985). In the 1990s, the development of the concepts of unit root and co-integration added twist to studies employing the causality test, see for example, Sharma et al. (1991), Ghartey (1993), Xu (1996), Riezman et al. (1996), Huang, Oh and Yang (2000). Broadly speaking, from all these articles it can be said that export growth can promote economic growth and vice versa. Economic theories suggest that higher exports mean more openness to trade, which implies higher productivity. Efficiency is aided further by production for international markets since this permits greater economies of scale and forces firms to hold down their costs in order to remain competitive in international markets. Additionally, profitable export industries stimulate additional investment, encourage an increased flow of new technology and stimulate increased consumption.

2) Debt

It is commonly argued that high levels of debt are associated with particularly large negative effects on growth. But, an influential series of papers by Reinhart and Rogoff (2010, 2012) argues that there is a threshold effect whereby debt above 90 percent of GDP is associated with dramatically worse growth outcomes. Otherwise, lower debt values have insignificant impact on GDP.

3) Foreign Aid

Foreign aid has a mixed impact on economic growth of developing countries and there is no robust evidence that aid affects growth. Of course, this does not imply that aid is necessarily ineffective. It has been seen empirically that foreign aid appears to have an adverse effect on economic growth in developing countries (Ekanayake). There had been attempts to estimate the causal effect of foreign aid in the past, all aiming to identify variables that affect growth only through aid and use them as instruments for aid. One group relies on variables that are based on the size of the recipient country's population to predict how much aid a country receives, Rajan and Subramanian (2008), a second uses past values of aid to predict current ones and estimates difference or system GMM regressions, Minoiu and Reddy (2010), and a third predicts aid with the recipient's political connections to their donors, Bjørnskov (2013). However none of these strategies meets their goals and it cannot be said convincibily if foreign aid has a positive or negative impact on a country's growth.

4) FDI

The next variable is foreign direct investment inflow into the country which is represented as FDI. The unit of FDI is percentage of GDP. It is the amount foreign citizens or companies invest in the local economy. Foreigners' investment creates jobs and additional economic activity that benefits the local economy. Thus, the predicted sign of the coefficient of FDI is positive. FDI can also facilitate 'agglomeration economies' through industry clustering and networking, and lowering costs for all producers in the market (Krugman 1991).

Pugel (2007) reports that FDI increases technological spillover benefits, widens the scope of international competition and strengthens the supply side capabilities of a host country for producing and selling goods and services, which lead to higher economic growth. Edwards (1992) also points out that a country with a higher degree of economic openness can grow faster by absorbing new technologies at a faster rate than a country with a lower degree of openness.

5) Labor Force

Previous studies show that the Gross Domestic Product (GDP) has a positive relationship with Labor Force Participation and Gross Fixed Capital Formation (Duval, Eris & Furceri 2010). They found the labor force participation hysteresis in industrial countries and used the impulse-response function approach to find the magnitude effect of labor force participation on industrial sector. Denton, Spencer (1997) using the trend analysis technique investigated the population, labor force and the long term economic growth and found positive effect too.

6) Gross Fixed Capital Formation

A number of economic theories have proved that capital formation plays a crucial role in the models of economic growth (Beddies 1999, Ghura, 1997). This give rise to the concept of Capital fundamentalism which embodies the belief that the rate of physical capital accumulation is the crucial determinant of economic growth. This can be viewed in the work of Youopoulos and Nugent (1976). Additionally the growth models developed by Romer (1986) and Lucas (1988), as discussed earlier in this article predicted that an increase in capital accumulation will result in a permanent increase in the growth rates. It is believed that capital formation has a strong positive impact on the economic growth as it determines the national capacity to produce, which in turn affects the economic growth.

7) Inflation

Though it's hard to estimate a direct impact of inflation on the economic growth but there are certainly indirect ways how inflation can effect a nation's economy. E.g. it can lead to an uncertainty about the future profitability of investment leading to more conservative investment strategies which lower the levels of investment and economic growth. Inflation may also reduce a country's international competitiveness, by making its exports more expensive. Moreover, inflation can interact with the tax system to distort borrowing and lending decisions. Dewan & Hussein (2001) found in a sample of 41 middle-income developing countries, that inflation was negatively correlated to growth. However, whenever there is expected inflation, governments around the world take appropriate steps to minimize the ill effects of inflation to a certain extent. Although it's true that inflation reduces the value of money and makes it difficult for the common people, its overall impact on the economic growth has been a topic of research in the past and present.

8) R&D Expenditure

It appears that there is a relatively strong positive effect of R&D expenses on all countries whether it's developed or developing. Gene et al (2010) investigated relations between R&D and economic growth for 34 countries from 1997-2008. Using the panel causality method they found out that there is a unidirectional causal relationship running from R&D to economic growth. Guloglu et al. (2012) too examined the causal relations among R&D expenditures and economic growth in 13 high income OECD countries, from 1991-2007.They estimated using a trivariate panel vector autoregressive model (VAR) employing panel fixed effects and Generalized Method of Moments (GMM) methods. They proved that relations between R&D and economic growth is positive and significant. Grocer (2013) studied R&D expenditures and its effect on high technology exports for 11 developing Asian countries from 1996-2012. They concluded that an increase of 1% in R&D expenditures raised the high technology export by 6.5%, the information-communication technology exports by0.6% and the economic growth by 0.43%.

9) GINI Index

The relationship between inequality and economic growth gives a mixed opinion and no strong conclusion can be drawn. Perrson (1991) and Alesina (1994), found evidence of a negative relationship between the two variables of interest. On the contrary Zou (1998) and Forbes (2000) find that greater inequality is associated with faster economic growth. Barro (2000, 2008) too claimed that inequality has a positive effect on GDP growth in advanced economies, but has a negative impact in the developing ones.

4. Methodology

At this stage now, I use the data as collected and discussed above, for data interpretation and analysis. I adopt the following process and strategy to get the results, which would be discussed in detail in the following paragraphs of this section.

1. Find the embedding parameters; "d"; time delay and "m"; dimension.
2. Compute the Recurrence Plot (RP) and RQA measures for each of the socio-economic factors.
3. Compute the Joint Recurrence Plot (JRP) and Cross Recurrence Plot (CRP) for the combinations of socio-economic factors with the Economic growth (GDP).
4. Analyze and understand the patterns and RQA values.
5. Conclude the findings.

1) On reconstructing the Phase Space

I assume that the given economic time series of any given variable is generated by a dynamical system. The specific state of this system can be represented by a point in the phase space and time evolution of the system creates a trajectory in the phase space. By doing so, I can consider my economic time series to be a projection of trajectory of dynamical system to one of the coordinates of phase space. This is enabled by formulating the Taken Theorem (Taken 1981) which says that it is possible to reconstruct the phase space from the time series. The phase space reconstruction is done by the method of time delay.

If we have a time series of a scalar variable, I construct a vector

$$x(t_i), \qquad i = 1, \ldots\ldots, N$$

in phase space in time t_i as following:

$$\boldsymbol{X}(t_i) = [x(t_i), x(t_i + d), x(t_i + 2d), \ldots\ldots, x(t_i + (m-1)d)]$$

where i goes from 1 to $N - (m-1)d$, d is the time delay, m is the dimension of reconstructed space and $M = N - (m-1)d$ is the number of points or states in the phase space. It is assumed that when the embedding is done in a proper way, dynamics reconstructed using this formula is equivalent to the dynamics on an attractor in the origin phase space in the sense that characteristic invariants of the system are conserved. I estimated the two parameters – time delay and embedding dimension, using the algorithms below.

2) On finding the Time Delay

I use the average mutual information (AMI), to determine the time delay which is a kind of generalization of autocorrelation function. Average mutual information between sets of measurements A and B is defined as

$$I_{AB} = \sum_{a_i b_j} P_{AB}\,(a_i, b_j) log_2 \left[\frac{P_{AB}(a_i, b_j)}{P_A(a_i)\, P_B(b_j)}\right]$$

where $P_A(a_i)$ is the probability of occurrence of (a_i) in set A, $P_B(b_j)$ is the probability of occurrence of (b_j) in set B and $P_{AB}(a_i, b_j)$ is the associated probability of co-occurrence of a_i in set A and b_j in B. When I use $x(t_1), x(t_2), \ldots\ldots,\ x(t_i)$ as set A and $x(t_1 + d), x(t_2 + d), \ldots\ldots,\ x(t_i + d)$ as set B, then the quantity

$$I(d) = \sum_{x(t_i)x(t_i+d)} P(x(t_i), x(t_i + d))(log_2 \left[\frac{P_{AB}(a_i, b_j)}{P_A(a_i)\, P_B(b_j)}\right]$$

represents the average information content that we know about value *x* in time *(t+d)* from value *x* in time *t*. As per Fraser (1986), the most appropriate estimation of the time delay *d* is at the location of the first local minimum of *I(d).*

3) On finding the Embedding Dimension

I used the concept of false neighbor to find the embedding dimension. A false neighbor is a point in the dataset that looks like a neighbor to another as the orbit is seen in a too small embedding space. To elaborate a bit, when the trajectory is projected to the space of too little dimension, trajectory crosses itself and so called false neighbor states occur. Now as the dimension of the phase space reconstruction increases, number of trajectory self-crossings and false neighbors decreases.

Let y(i) be a point in the reconstructed phase space, with $y(i)^r$ as its r^{th} nearest neighbor. I now compute the Euclidean distance L_2 between them as

$$Rm^2(y(i), y(i)^r) = \sum_{k=1}^{m-1} [y(i + kd) - y^r(i + kd)]^2$$

Now, increase m to m+1 and compute the new distance as $R^2_{m+1}(y(i), y(i)^r)$

$$R^2_{m+1}(y(i), y(i)^r) = Rm^2(y(i), y(i)^r) + [y(i + kd) - y^r(i + kd)]^2$$

The point $y(i)^r$ is called a false neighbor if the following condition holds true;

$$\left[\frac{R^2_{m+1}(y(i), y(i)^r) - Rm^2(y(i), y(i)^r)}{Rm^2(y(i), y(i)^r)}\right] > R_{tol}$$

Where R_{tol} is the predefined threshold. In the present research study I took $R_{tol} > 10$ as suggested by Kennel (1992). The percentage of false nearest neighbors (FNN) is computed for each m of a set of values. The embedding dimension "m" is found when the first percentage of FNN dropped to zero. The **Error! Reference source not found.** below shows the computation for calculating the embedding parameters for the inflation dataset. Note that the "m" value in this case is equal to 4.

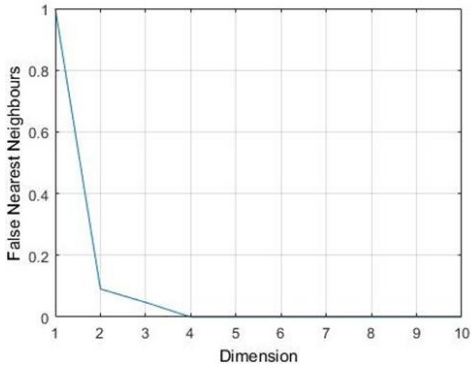

Fig. 1. Calculation of embedding dimension using False Nearest Neighbor technique

Once, the values of embedding parameters "d" and "m" are estimated I can calculate the RP and RQA for each of the dataset. In this view, please note that each delayed vector hence constructed along a period of (m-1)d years, for a given set of d and m values, represents the evolution of the time series during this period. This reconstruction of the phase space allow me to recognize changes at any appropriate time. The Recurrence Plots (RP) with their respective data set is a shown in Figure 2 - 11. Table 4 summarizes the RQA measures of each of the datasets. It can be seen that besides the RP of inflation and official aid received, the other RPs are not at all homogenous. In the next section, I will analyze the graphs and trends keeping in mind the important evets that occurred during this time. I would also try to see the impact of the 1990 FDI and disinvestment in India, which ended License Raj and opened up the economy for the world (Agihon 2006) and the 2008 global recession (Sher 2010) in these results. The recurrence plots, displayed in this work, allow us to study the recurrence of a state at a particular calendar date. Fixing a period or date on the horizontal axis, I am able to observe the recurrence of such events along the calendar dates on the vertical axis.

5. Results and Interpretation

1) Official aid received

In the past, India was increasingly relying on the western aid. In fact in the mid-1980s, India was one of the world's largest recipient of foreign aid. Now foreign aid constitutes less than 0.5 percent of the country's national GDP and has become marginal in overall economic development. This is the result of continuing Indian policy to oppose the country being perceived as a net recipient of foreign assistance. India has increasingly started to portray itself as a regional power and with its quest

for a permanent membership in United Nations Security Council, it has strove to reduce its dependence on external aid and prepay as much external debt as possible. This helped India downsizing its relationship with small donors, and maintain a cordial—but not fawning—relationship with large donors such as the World Bank, the Asian Development Bank, and Japan. As a result of these efforts, in 2003, India became a net creditor to the International Monetary Fund and the World Food Program after having been a borrower from these organizations for years. At this time, the official aid received, reached its minimum value in the past 30 years since 1970. This sudden irregularity in the pattern is observed as the big squares on the main diagonal of RP.

I don't find any strong relation and dependence of foreign aid received and the GDP. The RP of the official aid received doesn't show any pattern and any interdependence on the other socio-economic factors. E.g. it was expected that a pattern of increasing official aid should be homogenous with an increase in Gross fixed capital formation (infrastructure development) or Research & development expenditure or Central Government debt, but unfortunately no such relation can be concluded. Although I notice a butterfly like structure at the same time (1990) as in the GDP RP which shows some kind of inter-dependence between the two if not a strong one. The RQA measures of Recurrence Rate and entropy are 0.0854 and 0.6365 which shows a large amount of uncertainty with the data, i.e. the future values cannot be forecasted with much certainty on the basis of studying the current and past economic data.

2) Inflation

India's track record of inflation is good in the sense that it has never had to face the panic of hyperinflation, but having said that it is tumultuous in terms of fluctuations in inflation, witnessing relatively high rates on account of the supply shocks emanating mainly from agriculture and oil prices. Nationalization of fourteen private sector banks on July 20, 1969 was the single most important economic decision taken by the Govt. since 1947. Therefore years of 1973-74 and 1974- 75 were exceptional, as it was for the first time since independence that inflation during these two years crossed the 20% mark. I am however not able to notice this trend in the RP figure of inflation. The inflation recorded during these two years from 1973 – 1975 were at 20.2% and 25.2% respectively, due to additional events such as the failure of Kharif crops in 1972-73 also and the hike in crude oil prices in 1973. Another major event was in the years 2001- 04 where we witnessed moderation in inflation to 3.6% and 3.4%. This was because of the increase in administrated prices of fuel products and record public stocks of food grains with the Food Corporation of India (58 million tons in January 2002), together with active private trade that accorded stability to food grain prices and thus to the overall inflation. This sulk can clearly be seen on the RP diagonal in the form of big black blotch during the corresponding timeline. Inflation too, like official aid is an approximate stochastic process with low determinism measure of 0.1395 and laminarity of 0.3256. I don't find any strong interdependence of inflation on the GDP of India. However the time period of 2004 – 2010, during which the inflation increased due to 2008 global recession can be noticed on the RP as rare even occurrence in the form of black blotch on the time scale 32 – 38 respectively on RP axis.

3) Labor Force, GINI Index &R&D expenditure

All three of the factors, show a positive yet weak effect on the country's GDP. It is seen that all of them help increasing the GDP, but their effect is low and not so potent. This is a bit in contrast with the findings of Romero (1990), who said that the technology advancement is the prime factor and contributor to a nation's GDP. The constant value of labor force for year 2005-2010 can be determined in the RP as big black squares on the top right. For GINI index the affect is blurry and nothing can be concluded on the same. The value of recurrence rate (Labor Force = 0.0633, R&D expenditure = 0.0417, GINI = 0) for all the three are the lowest among all the factors. This shows highly deterministic and non-periodic nature.

4) Central Government Debt

I see a non-negative effect of the Govt. debt on the Indian GDP. It can be argued that this effect can be attributed to the fact that the debt % of GDP in all these years was around 50 – 60%, which is less than the 90% threshold value, as suggested by some researchers before. For example, Reinhart and Rogoff (2010) claimed that there is a particular threshold effect such that countries having debt above 90 percent of GDP have a growth performance dramatically lower than others. Cecchetti et al (2011) find that beyond 96 % of GDP, Govt. debt becomes a hindrance on the growth and hence countries with high debt must act quickly and decisively to address their fiscal problems. Rother (2013), focusing only on Euro area also find that debt has a non-linear effect on growth, leading to lower growth when it exceeds 95 percent.

B. Exports of goods and services, FDI and Gross Fixed Capital Formation

1) FDI

India had adopted a stringent foreign policy since 1960s to 1990. During this period the government adopted a selective and highly restrictive foreign policy as far as foreign capital, type of FDI and ownerships of foreign companies was concerned. The soaring oil prices continued low export and deterioration in balance of payment position during 1980s forced the government to encourage FDI and allow MNCs to operate in India. So 1990 was the turning point in the Indian history when it comes to FDI.

It can be seen that there has been a steady flow of FDI in the country up to 2004, but there is an exponential rise in the FDI inflows from 2005 onwards. The amount of FDI inflows in India during 2005- 2006, that included, equity capital, came to US$ 5.5 billion, which is an increase of around 72% in comparison with the previous year. The increase was largely due to the expanded list of industries or sectors which were opened up for foreign equity participation. This is aided by relaxation of various rules, regulations and introduction of various policies by the government to promote the FDI inflows. Both of these important events of 1990 and 2005, can be easily seen on the RP of FDI. While the first one is in the form of

square blotch on the diagonal line, the latter is of the shape of a butterfly.

There is a widely shared view that FDI promotes exports of host countries by (a) augmenting domestic capital for exports, (b) helping transfer of technology and new products for exports, (c) facilitating access to new and large foreign markets, and (d) providing training for the local workforce and upgrading technical and management skills. The positive impact of FDI on the other factors mentioned in this article namely GDP, GFCG can be assessed by overlapping their respective RPs and JRPs. I notice that the key patterns i.e. butterfly shape and square blocks on the diagonal line overlaps in all the three RPs, hence strongly supporting the statement above.

2) Exports

The exports growth in India, had been tremendous since the 1990 liberalization. In a time span of 20 years the exports value had increased four times. As we see in the RP of exports, there is a decreasing black area along the diagonal as we go from the lower left corner to the upper right. This signifies a strong increasing trend which can be justified by seeing the actual growth curve. The financial crisis that surfaced in the US in late 2008 led to a sharp contraction in world trade that was much greater than the fall in global output.

During 2010-14, India's export performance conformed to the average. Its share in world exports stayed in the range of 1.5%, while its share in developing countries' exports remained unchanged at 4%. India probably fared worse than the average in 2015. This was due to low crude oil prices and Competitive Prices of India Export Products. These small glitches in the increasing export trend for Indian economy in the year 2008 and 2014, unfortunately cannot be located in the RP. Nevertheless, exports and GDP, using RP and JRP shows a strong correlation between the two.

Exports growth affect poverty and income inequality both directly and indirectly. The direct effect operates through changes in factor incomes, which in turn depends on the composition of exports. If skill-intensive export items grow faster than relatively less-skill intensive exports, inequality among the wage earners accentuates, despite all of the wages going up. The increase in unskilled wage should, however, lower the incidence of poverty since most of the working poor belong to the class of unskilled workers. But once again, trade may asymmetrically affect poverty in rural and urban areas depending upon the relative growth in manufacturing and agricultural exports. The famous inverted-U relationship put forward by Kuznets (1955) indicates that output growth initially worsens income inequality but later lowers it as the opportunities for upward mobility that growth brings in spreads out more evenly across the different income classes. If we notice the GINI index and the exports graph, we see an increasing trend in the GINI coefficient from 1992 to 2010, during the time when exports showed a strong increasing trend too. However the GINI coefficient is on a decline since 2010. It is hard to comment if the decreasing trend is due to Kuznets theory or some other factor, and it would be interesting to research in this area.

3) Gross Fixed Capital Formation

GFCF gained momentum consistently through 1980s, due to sharp focus on Five-Year Plans. The impetus received in 1990s, primarily through liberalization led to a rise in investments, particularly in the past one decade. Contribution to GFC by the public sector remained highest from 1960s until 1996. Government contribution dominated total investments in the economy as India ushered in liberalization in the early 1990s. Spurred by economic reforms and a favorable business environment, private sector now leads in investments in the economy with 36% share in total capital formation. Private investments grew in 2000-10 and overtook household investments to become the largest contributor to total investment. During this period, GDP growth also rose to 9%. Although all economic growth cannot be attributed to investments, importance of capital formation remains paramount in economic development. This period from 2000 – 10, when the GCFC gained momentum and increased sharply can be seen in the RP in the form of square blocks on the diagonal. The post 1990 liberalization time period can also be observed as the same butterfly like shape on the diagonal as on the GDP RP. It can be said from the JRP and RP of GCFC and GDP that a positive capital formation can lead to sustainable long-term increase in the GDP. Although capital formation has increased in India since independence to touch 36% of GDP, it still remains below rates achieved in high-growth economies, such as China. Investment levels in China have risen to a high of 50% of GDP, which also underlines the high economic growth it has been able to sustain for the past 25 years.

6. Summary

To summarize, all the three factors; FDI, exports and GFCF have a strong positive effect on the GDP, and the same was depicted very well in the RP and JRP. The RP of exports shows decreasing large black area from the bottom left to the top right. This depicts a strong increasing trend. The same can be said looking at the trend RQA measure, trend (TT) for exports which is equal to 8.3906, in fact highest amongst all the other factors. Looking at all the three JRPs and RPs, I see a strong correlation and overlapping of the plots, especially in the time period of post 1990, and 2008 global crisis. The JRP depicts these two occurrences as rare events in the form of black spots between 20 – 25 and around 45 on the time scale on JRP axis respectively. All the three values also show large determinism (DET), recurrence rate (RR) and laminarity (LAM) values (> 0.9, in case of DET and LAM), which shows good recurrence results and better possibility to forecast and predict their values in the future. These quantification and high recurrence values are important because they help us to capture closely the dynamic interaction, and uncover the extent of coupling between two factors. When two processes interact, they adopt to each other, leading the response to exhibit recurrent states in RP and JRP.

The Cross Recurrence Plot (CRPs) of all the socio-economic factors with GDP is as shown in the Figure 12. However we can see that they provide misleading results and information and not suitable for this application. This is due to different dimensions

for the different socio-economic factors. The same result was also concluded by (Romano, 2004), asserting that the method of CRPs is not appropriate for the analysis of synchronization of oscillators. Joint Recurrence Plots (Figure 13) on the other hand do not mix the phase spaces of two different states. Furthermore, it consider a different and unique threshold for each system, so that we can apply the criteria to choose them separately. By doing so, the dimensionality mismatch problem mentioned before is also overcome, because the joint recurrence is still well defined, as well as in the case of having different physical units of each component.

To summarize, using the techniques of RP and RQA, on the past data of 49 years of Indian economy I can conclude that:

1. Exports of goods and services, Foreign Direct Investments and Gross Fixed Capital Formation have the strongest and the most positive impact on India's growth economy.
2. Inflation, Total Govt. debt and Official aid received have the least impact on the Indian economy growth.

To verify and cross examine my findings in this research article, I calculate the maximum correlation coefficient, of the six factors; the top and bottom three, with the Indian GDP using ACE algorithm (Breiman 1985). Alternating conditional expectation or ACE as it is commonly known is an algorithm for estimating the transformations of a response and a set of predictor variables in multiple regression. It produces the maximum linear effect between the (transformed) independent variables and the (transformed) response variable. These transformations can give the data analyst an insight into the relationships between these variables so that relationship between them can be best described and non-linear relationships can be uncovered. The results from this assessment demonstrate that ACE identifies the same accurate relationship between the different variables and the GDP growth as concluded by Recurrence Plot Theory. The Table 3 shows the maximum correlation coefficient thus calculated using ACE algorithm. It can be seen that our findings, is synonymous with the results. The three top factors FDI, Exports and Gross Fixed Capital Formation showed > 0.9 maximum coefficient, while the debt, aid and inflation showed lesser values.

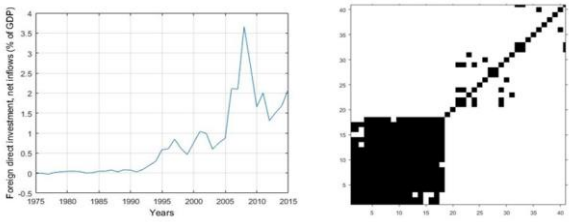

Fig. 2. GDP per capita (1967 - 2015) trend and its RP

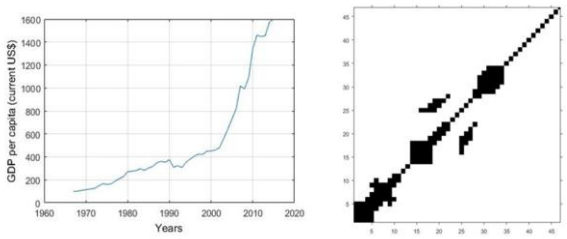

Fig. 3. FDI net inflows (1975-2015) and it's RP

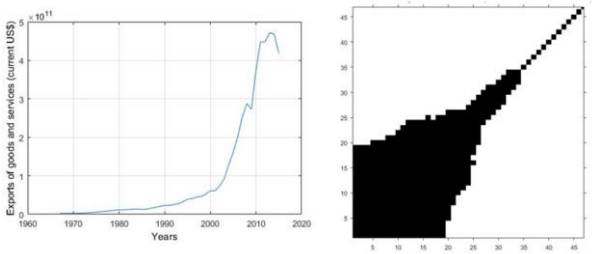

Fig. 4. Exports of goods and services (1967 - 2015) and RP. The arrow shape on the RP plot is the sign of a strong trend

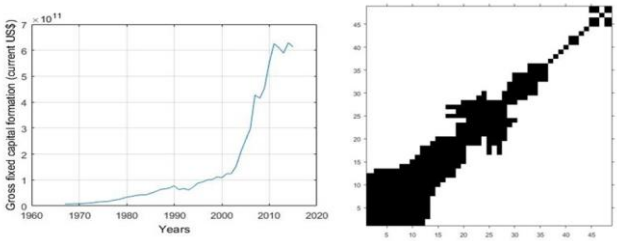

Fig. 5. Gross Fixed Capital Formation (1967-2015) and it's RP

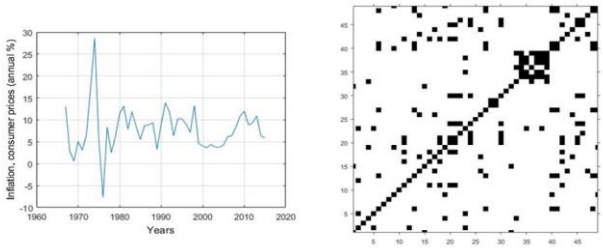

Fig. 6. Inflation (1967-2015) and its RP. The homogeneity of the RP plot reveals stationary.

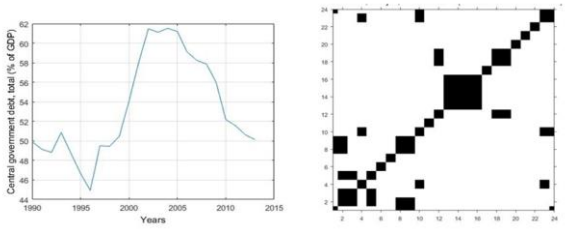

Fig. 7. Central Govt. Total debt (1990-2015) and it's RP

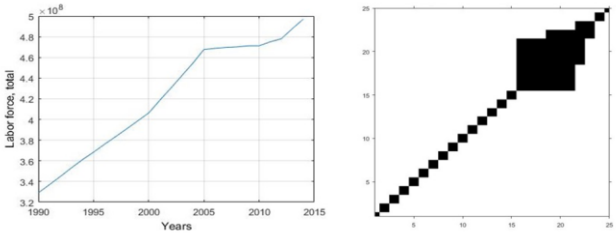

Fig. 8. Labor Force (1990-2015) and it's RP

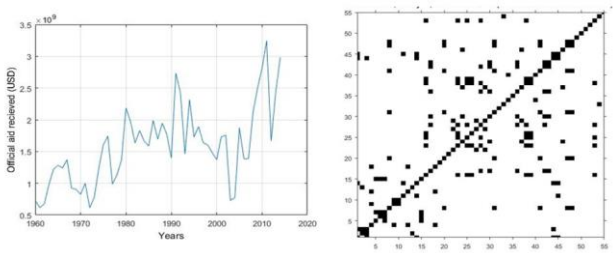

Fig. 9. Official Aid Received (1960-2015) and it's RP. The RP is homogenous, without any remarkable pattern, suggesting stationarity.

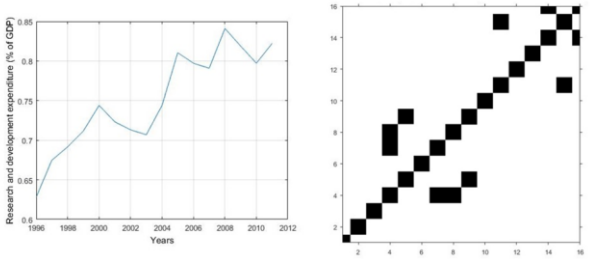

Fig. 10. R&D Expenditure (1995-2015) and it's RP

(RQA), to evaluate the patterns in the economic data. As seen from the results, this methodology provides a quick and easy assessment of the general health and key pattern in the economic data. Any rare events in the time series whether it's a surge (post 1990 liberalization), or sulk (2008 global crisis) can be observed and quantify using this method. Indeed from the RP we can comment on the stationarity and the behavior of the series when looking at the homogeneity and at the different shapes along the diagonal line. We can see that using RQA measures assessment, one can assess the time series, as being deterministic or complete random. Indeed if the RP does not look homogeneous and if black boxes or shapes are present, it can be argued that a rare event occurred. Moreover at the same time the RQA variables, in particular DET and ENT, take high values. In general the low values of these variables can be associated to stochasticity, while high values are associated with a deterministic behavior, as seen in Table 2. The inhomogeneity of RP and the high values of RQA variables reveal events that can be interpreted as a thermodynamic phase transition, if the homogeneity and low values are restored after this event.

In short, this method is capable of conveying all the information visually without any loss of information. This saves a lot of time, than the normal way of extracting features from time and frequency domain and analyzing each one of them separately.

Table 3
Key socio-economic factors and their Maximum correlation coefficient

	GDP & Exports	**GDP & FDI**	**GDP & Gross Fixed Capital Formation**	**GDP & Inflation**	**GDP & Total debt**	**GDP & Official Aid**
Maximum Correlation Coefficient	0.99543	0.96952	0.99827	0.68162	0.86478	0.79654

Table 4
RQA Variables for the different socio-economic factors

Variable	**RR**	**DET**	**L**	**ENTR**	**LAM**	**TT**	**RTE**	**T^2**
GDP per capita	0.0564	0.9344	4.3846	1.7782	0.7787	2.7941	0.7081	3.9630
Exports of goods and services	0.2498	0.9963	15.8235	2.7517	0.9944	8.3906	N/A	7.4667
Central Government Debt	0.0833	0.3043	2.3333	0.6365	0.5217	2.1818	0.5921	8.2308
Gross Fixed Capital Formation	0.1599	0.9309	9.2105	2.4062	0.9601	4.9452	0.5390	5.1143
Foreign Direct Investments	0.1915	0.9045	8.3529	2.4255	0.9236	7.2500	0.6835	7.7500
Inflation, consumer prices	0.0731	0.1395	2.4000	0.5004	0.3256	2.2400	0.8028	10.5833
Labor Force	0.0633	0.9474	4.5	1.3863	N/A	N/A	N/A	4.0
Research & Development Expenditure	0.0417	0.4	2.0	0	0.2	2	0.7178	N/A
GINI Index	0	N/A	N/A	N/A	N/A	N/A	N/A	N/A
Official aid recieved	0.0552	0.0854	2.3333	0.6365	0.25	2.05	0.6766	9.7742

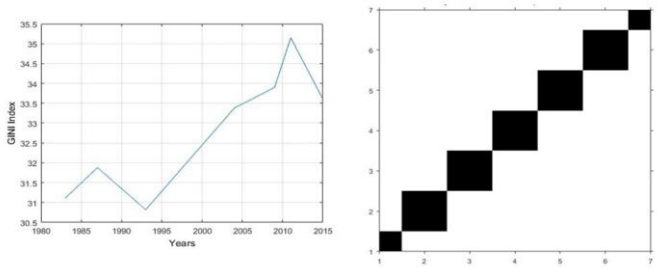

Fig. 11. GINI Index (1984-2015) and it's R

7. Conclusion

There were more than one intentions and objectives before the start of this research and we conclude them based on the results and findings as below:

First, the prime objective was to test the credibility of Recurrence Plot (RP) and Recurrence Quantitative Analysis

Second, I wanted to see if by using the RP and JRP techniques, is it possible to comment on the interdependence between GDP and other socio economic factors and to find out the most important factors which plays a dominant role in the economic growth of a country. I find out that by analyzing the overlapping behavior of different RP and JRPs, we are able to comment on the interdependence among different factors. This method of assessment is quick and handy. I observed that FDI, Exports and Infrastructure Development are the three most important key factors in the economic growth of India while inflation, Govt. debt and official aid are the least impactful for the Indian economy growth. In order to validate the results from this research, I used ACE regression algorithm to compute the maximum correlation coefficient of all the three factors.

Incidentally, the results from both the methods are similar and points out to the same result. These findings are consistent with previous research and these factors have the same effect on economic growth of both developed and developing countries.

In conclusion, the possibility to use the RP and RQA methods in order to detect the key factors and the patterns in the economic development of a country has been investigated. It has been shown that, RP methods present an easy and quick assessment and analysis to detect a difference in state and recognize the critical regime. This study has also political implications on how to effectively raise the economic conditions of India, however research with more data set needs to be done. The policy suggestions generated by such research could have a significant impact on the growth rates of India.

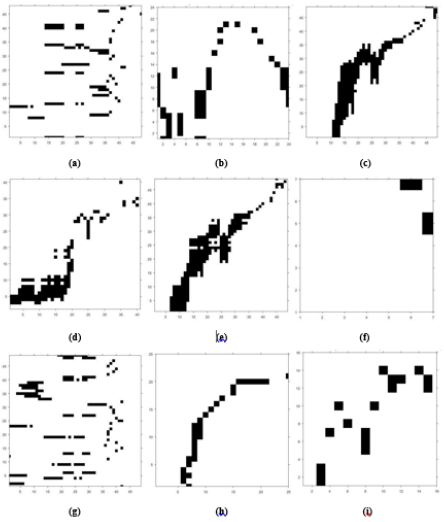

Fig. 12. Cross Recurrence Plots of GDP with (a) Official Aid (b) Govt. debt (c) Exports (d) Foreign Direct Investments (e) Gross Fixed Capital Formation (f) GINI Index (g) Inflation (h) Labor Force (i) R&D expenditure

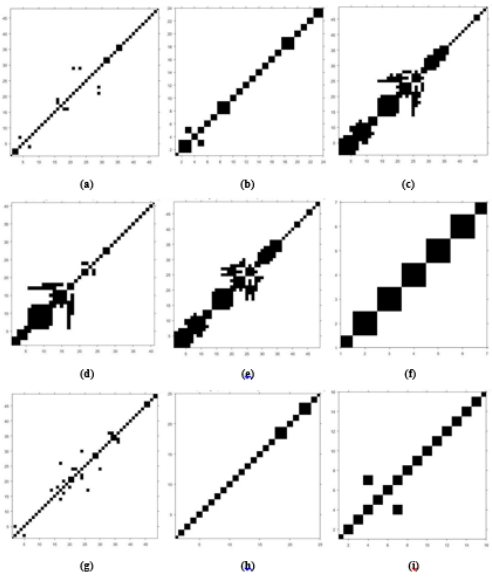

Fig. 13. Joint Recurrence Plots of GDP with (a) Official Aid (b) Govt. debt (c) Exports (d) Foreign Direct Investments (e) Gross Fixed Capital Formation (f) GINI Index (g) Inflation (h) Labor Force (i) R&D expenditure

References

[1] Alesina, A., and D. Rodrik. 1994. Distributive Politics and Economic Growth. The Quarterly Journal of Economics, Vol. 109, No. 2, 465-490

[2] Agihon, P. (2006),The Unequal Effects of Liberalization: Evidence from Dismantling the License Raj in India Philippe Aghion, Robin Burgess, Stephen Redding, and Fabrizio Zilibotti NBER Working Paper No. 12031 February 2006

[3] Barro, R. J. 2008. Inequality and Growth revisited. Working paper series on regional economic integration, n. 11.

[4] Barro, R. J. 2000. Inequality and Growth in a Panel of Countries. Journal of Economic Growth, 5, 5-32

[5] Barro, Robert J. (1996). Determinants of Economic Growth: A Cross-Country Empirical Study. National Bureau of Economic Research, Working Paper: 5698.

[6] Baum, Anja, Cristina Checherita and Philipp Rother, 2013, "Debt and Growth: New Evidence from the Euro Area", Journal of International Money and Finance, Vol. 32, pp. 809–21.

[7] Beddies (1999); "Investment, Capital Accumulation and Growth: Some Evidence from Gambia: 1964-1998"; IMF Working Paper 99/117, August.

[8] Breiman L, friedman, J.H. , Estimating Optimal Transformations for Multiple Regression and Correlation, 1985 American Statistical Association Journal of the American Statistical Association September 1985, Vol. 80, No. 391, Theory and Methods

[9] Bjørnskov, C (2013), "Types of Foreign Aid", University of Aarhus Working Paper No. 2013-08.Bazzi, S, and M A Clemens (2013), "Blunt Instruments: Avoiding Common Pitfalls in Identifying the Causes of Economic Growth", American Economic Journal: Macroeconomics 5, 2: 152-86.

[10] Cass, D. (1965), "Optimum growth in an Aggregative Model of Capital Accumulation", The Review of Economic Studies, Vol. 32, No. 3 (Jul., 1965), pp. 233-240

[11] Cecchetti, S. G., Mohanty, M.S., and F. Zampolli, 2011, "The Real Effects of Debt", Federal Reserve Bank of Kansas City, Economic Symposium 2011: Achieving Maximum Long-Run Growth, pp. 145–96.

[12] Chow, P.C.Y. (1987), "Causality between Export Growth and Industrial Development," Journal of Development Economics, 26, 55-63.

[13] Denton, F. T., Spencer, B. G., (1997). Population, labour force and long-term economic growth. Hamilton, Ont.: Research Institute for Quantitative Studies in Economics and Population, McMaster University.

[14] Dewan, E and S. Hussein. (2001). "Determinants of Economic Growth", Working Paper, Reserve Bank of Fiji

[15] Duval, R., Eris, M., & Furceri, D. (2010). Labour Force Participation Hysteresis in Industrial Countries: Evidence and Causes

[16] Eckmann J.-P., Rev. Mod. Phys., 53 (1981) 643

[17] Eckmann J.-P., OLIFFSON KAMPHORST S., RUELLE D. and CILIBERTO S., Phys. Rev. A, 34 (1986) 4971.

[18] Edwards, S. (1992a). Trade orientation, distortions and growth in developing countries. Journal of Development Economies, 39, 31-57.

[19] E. M. Ekanayake, Dasha Chatrna, "The effect of foreign aid on economic growth in developing countries", Journal of International Business and Cultural Studies

[20] Forbes, K. J. 2000. A Reassessment of the Relationship between Inequality and Growth. The American Economic Review, Vol. 90, No. 4, 869-887. http://www.jstor.org/stable/117312 (17/02/2014)

[21] Fraser, A.M. (1986), "Independent coordinates for strange attractors from mutual information", Physical Review A, Vol 33, Number 2

[22] Ghura, D. (1997), "Private Investment and Endogenous Growth: Evidence from Cameroon", IMF Working Paper 97/165, December

[23] Ghartey, E.E. (1993), "Causal Relationship between Exports and Economic Growth: Some Empirical Evidence in Taiwan, Japan and the U.S.," Applied Economics, 25, 1145-1152

[24] Gene, Murat Can & Atasoy Yesim (2010), The relationship between R&D expenditure and economic growth: Panel Data Analysis, The journal of knowledge Economy and Knowledge Management, V,Fall, 27-34

[25] Gocer, Ismet (2013), Effects of R&D expenditures on high technology exports, balance of foreign trade and economic growth. Maliye Dergisi 165, 215-240

[26] Guloglu, B & Tekin, R Baris (2012), A panel causality analysis of the relationship amomng research and development, innovation and economic growth in high income OECD countries, Eurasian Economic Review, 2(1), 32-47.

[27] Huang, B.N., J.J.R. Oh, and C.W. Yang (2000), "On the Causality of Exports and Economic Growth: An Application of Direct and Indirect Causality," Working Paper.
[28] Jung, W., and P. Marshall (1985), "Exports, Growth and Causality in Developing Countries," Journal of Development Economics 18, 1-12.
[29] Kennel, M., R. Brown, H. Abarbanel, 1992. Determing embedding dimension for phase space reconstruction using a geometrical construction, Phys Rev A 45 3403-3411.
[30] Krugman, P. (1991a). Geography and Trade. Leuven University Press: Leuven.
[31] ---------- (1994b). The myth of Asia's miracle. Foreign Affairs, 62-78.
[32] Koopmans, Tjalling C. (1965): "On the concept of optimal economic growth," in (Study Week on the) Econometric Approach to Development Planning, chap. 4, pp. 225–87. North-Holland Publishing Co., Amsterdam
[33] Lucas, RE.J. (1988), "On the Mechanics of Economic Development", Journal of Monetary Economics, 22: 3— 42.
[34] Zou, H., Li, H., 1998. Income Inequality is not Harmful for Growth: Theory and Evidence. Review of Development Economics, 318-334
[35] M. D. McKenzie, 2001. Chaotic behavior in national stock market indices. Global Finance J. 12 35-53.
[36] Michaely, M. (1977), "Exports and Growth: An Empirical Investigation," Journal of Development Economics, 4(1), March, 49-53.
[37] Michalopoulos, C., and K. Jay (1973), "Growth of Exports and Income in the Developing World: A Neoclassical View," A.I.D. Discussion Paper No. 28 (Washington, DC).
[38] Minoiu, C., Reddy, S. (2010), Development aid and economic growth: A positive long run relation, The Quaterly Review of Economics and Finance, 2010, Vol. 50, Issue 1, Pages 27-39
[39] Marwan, N., M. Carmen Romano, Marco Thiel, Jürgen Kurths (2007), Recurrence plots for the analysis of complex systems, Physics Reports 438 (2007) 237 – 329
[40] Marwan N, A. Groth, J. Kurths: Quantification of Order Patterns Recurrence Plots of Event Related Potentials, Chaos and Complexity Letters, 2(2/3), 301–314p. (2007)
[41] Marwan N, J. Kurths: Line structures in recurrence plots, Physics Letters A, 336(4–5), 349–357p. (2005).
[42] Marwan N, M. Thiel, N. R. Nowaczyk: Cross Recurrence Plot Based Synchronization of Time Series, Nonlinear Processes in Geophysics, 9(3/4), 325–331p. (2002
[43] Pugel, A. T. (2007). International Economics. McGraw-Hill Irwin: New York, USA.
[44] Persson, T., and G. Tabellini. 1991. Is inequality harmful for growth? Theory and evidence. NBER Working Paper No. 3599.
[45] Poincare, H Acta Math. 13, 1 (1890).
[46] Ramsey, F.P., (1928), "A Mathematical Theory of Saving", The Economic Journal, Vol. 38, No. 152 (Dec., 1928), pp. 543-559
[47] Romer , P. (1990), "Endogenous Technological Change", Journal of Political Economy, 98 (5), S71-S102
[48] Rebelo, Sergio T. (1991): "Long-Run Policy Analysis and Long-Run Growth," Journal of Political Economy, 99(3), 500–521.
[49] Reinhart, Carmen M. and Kenneth S. Rogoff, 2010, "Growth in a Time of Debt", American Economic Review: Papers & Proceedings, Vol. 100, No. 2, pp. 573–78.
[50] Reinhart, Carmen M., Vincent R. Reinhart, and Kenneth S. Rogoff, 2012, "Public Debt Overhangs: Advanced-Economy Episodes since 1800", Journal of Economic Perspectives, Vol. 26, No. 3, pp. 69–86
[51] Riezman, R.G., C.H. Whiteman, and P.M. Summers (1996), "The Engine of Growth or Its Handmaiden? A Time Series Assessment of Export-Led Growth," Empirical Economics, Vol. 21, No. 1, 77-110
[52] Romano. M.C., "Synchronization Analysis by means of Recurrences in Phase Space". PhD Thesis (2004), University of Potsdam.
[53] Raghuram G. Rajan & Arvind Subramanian, 2008. "Aid and Growth: What Does the Cross-Country Evidence Really Show?," The Review of Economics and Statistics, MIT Press, vol. 90(4), pages 643-665, 06
[54] Rajan, R G and A Subramanian (2008), "Aid and Growth", Review of Economics and Statistics 90, 4: 643-665.
[55] Sher, V. (2010), The Great Recession of 2008-2009: Causes, Consequences and Policy Responses, IZA Discussion Paper No. 4934 May 2010
[56] Solow, R.M., (1956), "A Contribution to the Theory of Economic Growth", QJE.
[57] Swan, T.W., (1956), " Economic Growth and Capital Accumulation", Economic Record, Vol 32, Issue 2
[58] Strozzi, F., J. M. Zaldivar, J. P. Zbilut, 2002. Application of non-linear time series analysis techniques to high frequency currency exchange data. Phys. A 312 520-538.
[59] Sharma, S.C., M. Norris, and D.W. Cheung (1991), "Exports and Economic Growth in Industrialized Countries," Applied Economics, 23, 697-708
[60] Takens, F. (1981). "Detecting strange attractors in turbulence", Lecture Notes in Mathematics, Springer, Berlin 898 366-381.
[61] Trulla, L., A. Giuliani, J. P. Zbilut, C. L. Webber, Jr.: Recurrence quantification analysis of the logistic equation with transients, Physics Letters A, 223(4), 255–260p. (1996)
[62] UK Essays. November 2013. The Indian Economy and The Licence Raj Economics Essay. [online]. Available from: https://www.ukessays.com/essays/economics/the-indian-economy-and-the-licence-raj-economics-essay.php?cref=1 [Accessed 14 March 2017].
[63] Webber, C. L., J. P. Zbilut, 1992. Embedding and delays as derived from quantification of Recurrence Plot, Phys Lett A 171 199-203
[64] Xu, Z. (1996), "On the Causality between Export Growth and GDP Growth: An Empirical Reinvestigation," Review of International Economics, 4(2), 172-84.
[65] Yotopoulos, P. and J. Nugent. 1976. Economics of Development: Empirical Investigations, New York: Harper and Row.