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Estimation of Ground Station Coordinates Using Normal Equation Stacking Method

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***Abstract*: The estimation of ground station coordinates has been accomplished for the Galileo navigation system. In the estimation process, measured data from multiple frequencies, satellite state vectors and satellite clock errors are taken as input and produce an estimate of station coordinates parameters. The ionosphere error-free code measurement data for dual-frequency and triple-frequency are used for the estimation of ground station coordinates. The present paper deals with the determination of station coordinates using the normal equation stacking method and solutions are found at each epoch. All possible combinations for dual-frequency and triple-frequency are analysed for the best combination for the station position estimates. Results and analysis are demonstrated using the methods described for the station's position estimates for BRUX station for Day of Year (DOY) 283, 2021. The estimated position coordinates are compared with precise station coordinates to elucidate the accuracy. The maximum accuracy achieved for the position estimates is 69 cm. It has been found that (E1, E5ab) is best among dual-frequency combinations and (E1, E5b, E5ab) is best among all frequency combinations. The use of triple frequency and normal equation stacking method improves the station's position estimates compare to dual-frequency. Results demonstrate the use of the Normal equation stacking method for coordinate estimates.**

***Keywords*: Point Positioning, Linear combinations, Normal equation stacking, least square method.**

1. Introduction

Global Navigation System is a system that consists of various navigation systems. Some of these include GPS-United States, GLONASS-Russia, Galileo-Europe, BeiDou-China, QZSS-Japan, and NavIC-India. The NavIC is divided into three segments: the space segment, the ground segment and the user segment [3,6]. The space segment consists of 7 IRNSS satellites [1, 2]. The ground segment of the IRNSS system comprises IRIMS (IRNSS Range and Integrity Monitoring Stations), the IRCDR ground station, and the IRSCF (IRNSS Spacecraft Control Facility) master control station and 2 INC (ISRO Navigation Centre) [3,7]. Each of the IRIMS ground stations is equipped with high fidelity receivers and Hydrogen-Maser/Cesium/Rubidium atomic clocks. The user segment of the system consists of single and dual-frequency users. It uses L5 and S frequencies and has an accuracy of 10 meters in the primary service region.

The Navigation systems like GPS started with the two frequencies. In the later stage, it has been found the utilization of third frequency as well. Several systems like GLONASS, GALILEO, BeiDou, having more than three frequencies at the early stage itself. Now, GPS also has a triple-frequency for several satellites. Basically, in the last decade, the use of triple-frequency become popular in the satellite navigation system. All GNSS system is in the process of having multiple frequencies. Because the introduction of third frequency improves the estimation of ionosphere delay and cycle slip in GPS (Global Positioning System), GLONASS and BeiDou satellite navigation system. The details of transmitting signals on several frequencies for Galileo, GPS and NavIC Navigation systems are provided in Table 1.

Table 1
Carrier frequencies and wavelengths

Navigation System	Signal	Frequency [MHz]	Wavelength (m)
Galileo/GPS/NavIC	E5a/L5	1176.45	0.2548
Galileo	E5ab	1191.795	0.2515
Galileo	E5b	1207.14	0.2483
GPS	L2	1227.60	0.2442
Galileo	E6	1278.75	0.2344
Galileo/GPS/NavIC	E1/L1/L1C	1575.42	0.1903
NavIC	S	2492.028	0.1203

Generally, the ground station position is computed using the following methods: The ordinary least square method and the Batch least square method. The ordinary least square method takes into account the observations made for a given epoch and then produces a new estimated ground station position. The Batch least square method estimates the position of a ground station after taking into account the observations made over a a fixed interval of time. In this article, the concept of station coordinates is solved through the normal equation stacking method. In this method, solutions are available at each epoch.

The present work has been divided into 5 sections. Section 2 is devoted to the theory of code measurement and the ionosphere error-free combinations. Section 3 of this article describes the steps to calculate the satellite state vectors and the satellite clock error at the time of transmission. It also includes the steps to estimate the ground station position. Section 4 of this article describes the results of the algorithms used to estimate the position of the BRUX station for the Galileo navigation system. Results of estimated position compared with precise station coordinates. Section 5 of this article describes the major findings of the algorithm.

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2. Theory

The pseudorange is computed by multiplying the velocity of light with the travel time of the signal and is expressed as

$$P_R = c\,dt = c\left(t_R - t_T\right) \tag{1}$$

Where P_R is the pseudorange, c is the speed of light (299,792,458 [m/s]) in a vacuum, $dt=\left(t_R - t_T\right)$ is the travel time of the signal[s], t_R is the time of observation in the receiver time frame [s], t_T is the time of transmission in the satellite time frame [s]. The time difference between the reception time of a given signal and its transmission time is known as the travel time of the signal. The traveling of the transmission signal from the satellite to a receiver passes through the atmosphere causes propagation delays (due to ionosphere and troposphere), ephemeris errors (due to relativity effects), clock errors (due to receiver and satellite clocks) and other systematic (e.g., hardware delays), random (measurement noise) and non-random (e.g., multipath delays) biases [4,5,8,9]. So, the expression P_i for the code measurement is as follows:

$$\begin{aligned} P_i = {} & G_R + c\,rclk - c\,sclk \\ & + I_{P_i} + Tr + relative \\ & + rhwd_{P_i} - shwd_{P_i} - sant \\ & + mpath_{P_i} + noise_{P_i} \end{aligned} \tag{2}$$

Where P_i is the observed code measurement [m] for f_i frequency, i (=1,2,3) is for multiple frequencies as mentioned in columns 2 and 3 of Table 1, G_R is the geometric range [m] and is defined as the geometric distance between the phase centers of the satellite and receiver antennas, $rclk$ is the receiver clock error [s], $sclk$ is the satellite clock error [s], I_{P_i} is the ionospheric delay at P_i code measurement [m], Tr is the tropospheric delay[m], $relative$ is the relativistic effect [m], $rhwd_{P_i}$ is the receiver hardware delay for P_i code measurement [m], $shwd_{P_i}$ is the satellite hardware delay for P_i code measurement [m], $sant$ is the satellite antenna phase center, $mpath_{P_i}$ is the multipath delay for P_i code measurement [m], $noise_{P_i}$ is the measurement noise for P_i code measurement [m]. The symbols used in Eqns. (1) and (2) are the same throughout this article.

The calculations of phase measurements are similar to code measurements except for the cycle slip occurrence and ionosphere delay. The difference in the ionosphere is of opposite sign as mentioned in Eq. (3). So, the equation for L_i phase measurement [m] for f_i frequency is

$$\begin{aligned} L_i = {} & G_R + c\,rclk - c\,sclk \\ & - I_{L_i} + \lambda_i N_i + Tr \\ & + relative + rhwd_{L_i} \\ & - shwd_{L_i} - sant + noise_{L_i} \end{aligned} \tag{3}$$

Where $\lambda_i\left(=C/f_i\right)$ is the wavelength [m] for f_i frequency as mentioned in the last column of Table 1, N_i is the cycle slip ambiguity at f_i frequency.

The ionosphere delay at code and phase measurements can be determined with the help of the linear combinations for dual and triple frequency. The linear combination of dual-frequency can be expressed as follows:

$$IF_P = \left(aP_1 + bP_2\right) \tag{4}$$

Where IF_P ionosphere free code measurements and the coefficients is are $a = f_1^2 / f_1^2 - f_2^2$, $b = -f_2^2 / f_1^2 - f_2^2$. The coefficients values and noise factor for ionosphere error-free dual-frequency combinations are tabulated in Table 2.

The triple-frequency ionosphere error-free combinations can be expressed as follows:

Table 2
Coefficient values and Noise factors for dual and triple frequency

f1	f2	f3	a	b	c	Noise factor
E1	E5b		2.422	1.422		2.8086
E1	E5ab		2.338	1.338		2.6938
E1	E5a		2.2606	1.2606		2.5883
E1	E5b	E5a	2.2985	-0.4402	-0.8582	2.6333
E1	E5ab	E5a	2.2857	-0.4323	-0.8534	2.6186
E1	E5b	E5ab	2.3245	-0.4452	-0.8792	2.6631

$$IF_P = \left(aP_1 + bP_2 + cP_3\right) \tag{5}$$

The coefficients of ionosphere error-free for triple-frequency combinations found using the exact solution as described in [9]. Several combinations can be formed for dual and triple-frequency combinations. The coefficients values and noise factor for ionosphere error-free dual and triple-frequency combinations are tabulated in Table 2.

The noise factor is presented in the last column of Table 2. It can be observed that the minimum noise factor is for (E1, E5a) in dual-frequency combination and (E1, E5ab, E5a) in triple-frequency combination. It is also related to the noise of the code and phase measurements. The total noise of the code/phase measurements is computed by multiplying the noise factor with the standard deviation of the noise. For example, if the noise of the code measurement (E1, E5a) is 25cm then the total noise for the (E1, E5a) will be 64.71cm. Similarly, the noise will be computed and results are given in section 4.

3. Algorithm

In the present article, the position of a station is estimated using precise satellite state vectors and satellite clock errors for dual-frequency and triple-frequency combinations. The code and phase measurements of different frequencies are collected from the RINEX database [14]. These data are archived daily [14]. The data in the RINEX file is 2880 epochs with an interval of 30 seconds. The code and phase measurements are then analyzed for nonzero observed values. These observed values are then analyzed for SNR values. For good signal strength, the threshold is 30 [page 27, 11]. The calculations are performed at an interval of 30 seconds. The present article deals with the estimation of station coordinates for dual-frequency and triple-frequency. The user's position is determined using the principle of Trilateration, which requires at least four satellites to determine the exact position of the station. Two algorithms are used to estimate the user's position and the receiver's clock error.

1) Computation of satellite state vectors and satellite clock errors at the time of transmission

The first algorithm is to determine the satellite's state vector in the ECEF (Earth Centered Earth Fixed) frame of reference and the clock error at the time of transmitting. The following steps are used for the calculation of satellite data (i.e., state vectors and clock errors):

1. In the first step of the calculation, the position and clock error of the satellites are obtained from precise ephemeris files of CDDIS [14]. The archived data are updated daily and archived for 288 epochs with an interval of 5minutes. The data required for the calculation of a user's position are of three days including the starting and end dates of the measurements, previous days and next days. So, at the starting and end epochs of the measurements, satellite data will be available. The position $\left(x_s, y_s, z_s\right)$, velocity and clock error at transmission time are obtained using Lagrange interpolation.
2. The calculation of the time of transmission is carried out by taking into account the following components: the receiver time, the ionosphere free pseudorange, troposphere delay, receiver clock error, satellite clock error, relativity, hardware delays and satellite antenna phase center.

$$t_{R_{new}} = t_R - IF_P/c + rclk + relativity + Tr/c + rhwd_P - shwd_P - sant \qquad (6)$$

The satellite clock error is computed at a time $t_{R_{new}}$ using Lagrange's interpolation. Then the transmission time is computed as:

$$t_T = t_{R_{new}} - sclk\left(t_{R_{new}}\right) \qquad (7)$$

3. The satellite clock error is corrected for relativity correction. The satellite's state vectors and satellite clock error are obtained at the time of transmission using Lagrange's interpolation.
4. The time difference between the transmission and the reception time is computed and referred to as travel time (*dt*).
5. Finally, the satellite position is corrected for Earth's rotation using the following formulae:

$$\begin{bmatrix} x_{S_{new}} \\ y_{S_{new}} \\ z_{S_{new}} \end{bmatrix} = \begin{bmatrix} \cos(\Omega) & \sin(\Omega) & 0 \\ -\sin(\Omega) & \cos(\Omega) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_S \\ y_S \\ z_S \end{bmatrix} \qquad (8)$$

Where $\Omega\left(=\omega_e\, dt\right)$ the angle due to Earth's rotation is, $\omega_e = 7.292115\times10^{-5}\ rad/\mathrm{sec}$ denotes Earth's rotation rate and *dt* is the travel time of the signal, $\left(x_{s_{new}}, y_{s_{new}}, z_{s_{new}}\right)$ is the new position of the satellite after the Earth's rotation.

2) Estimation of station coordinates

The following inputs are available for estimating the position of the station. They include ionosphere error-free code measurements, satellite state vectors and satellite clock errors. The following steps are followed by the algorithm to estimate the position of the GNSS station:

1. If no initial guess is available for the position's estimates, then Earth's center coordinates (0,0,0) are taken as an initial guess $\vec{x}_{init} = \left(x_{R_{init}}, y_{R_{init}}, z_{R_{init}}\right)$.
2. Use the previous algorithm to calculate the satellite clock error and the satellite state vectors.
3. Calculate the residue at each epoch:

I) The nonzero receiver's position is corrected for the two corrections: receiver's antenna offset and tide correction.

II) Compute the satellite antenna phase center and new satellite state vector corresponding to the satellite antenna phase center.

III) Use the satellite's position to calculate the geometric range of all visible satellites:

$$G_{R_j} = \sqrt{\left(x_R - x_{S_j}\right)^2 + \left(y_R - y_{S_j}\right)^2 + \left(z_R - z_{S_j}\right)^2}, \qquad (9)$$

Where $\left(x_R, y_R, z_R\right)$ is the coordinates of the Receiver's position and $\left(x_{S_j}, y_{S_j}, z_{S_j}\right)$ is the position of all visible satellites with indexing *j*=1, 2, 3... for the satellite.

IV) The pseudo range is corrected for the troposphere delay [10] and relativity effects [12].

V) The mean of the residue $\left(\mathrm{Res}_j = P_j - P_{C_j}\right)$ is computed at each epoch and filtered using two criteria. If the residue difference with computed mean exceeds 1 meter at least for one line of sight, then it goes for the second criterion. In the second criterion, residues pass through the MAD (Median Absolute Deviation) filter and the formulation of MAD is used as follows:

$$\left|\mathrm{Res}_j - median\left(\mathrm{Res}_j\right)\right| < M \times MAD\left(\mathrm{Res}_j\right) \times 1.4826 \qquad (10)$$

Where M is a factor and is found as 2. Because for $M=1$ throwing most of the line-of-sight residues. So, very few satellites remain for further calculation. $M=2$ satisfies the requirements of further calculation.

The above two criteria remove the outlier present in the residue before estimation. This further improves the position estimates.

4. The position and receiver clock bias of the station are estimated for the first valid epoch using the iterative least square method until the convergence is not achieved i.e., norm error<10^{-8} or the maximum number of iterations is 10.
5. In the subsequent valid epoch, the new estimates of position and receiver clock bias/error are derived from the normal equation stacking method [13].

4. Results and Analysis

Section 2 and 3 introduce the method used to estimate the receiver clock error/bias and station coordinates of GNSS stations. So, the methodology is applied on one of the GNSS stations namely BRUX for DOY 283, 2021. GNSS is a network of navigation systems that include GPS, Galileo, Glonass, NavIC, and QZSS. Out of these navigation systems, Galileo contains multiple frequencies in the measurement data. So, it takes advantage of the multiple frequencies in Galileo's data. Further several dual-frequency and triple-frequency combinations can be made to choose the best combination for position accuracy. Since for E6 frequency, service is closed [15]. So, E6 is not used for further analysis. A total of 3 combinations are used for dual-frequency. These combinations are (E1, E5b), (E1, E5ab), and (E1, E5a). The first observation type in the braces is used as frequency one and the second one is used as frequency two. These combinations are made only with the highest frequency because of the low noise factor for ionosphere error-free combinations as mentioned in Table 2. Total three combinations are made for triple-frequency. These combinations are (E1, E5b, E5ab), (E1, E5b, E5a), and (E1, E5ab, E5a). The first observation type in the braces is used as f1, the second observation type is used as f2, and the third observation type is used as f3.

The concept of Trilateration states that 4 visible satellites are needed to know the location of a user/ground station. The visibility of the satellites along with their geometry also contributes to the accuracy of the data. The geometry of a satellite is measured by the Geometric Dilution of Precision (GDOP). It has been observed that with 4 visible satellites, the position error can be bad. The following conditions/constraints are required to achieve good position accuracy:

1. The minimum number of visibility satellites is 4.
2. The minimum elevation angle is 5 degrees.
3. The minimum value of SNR is 30 dbHz.

The above conditions/constraints are holding good for all frequencies combinations. Although the minimum number of satellites is 4 but to achieve good accuracy and to maintain the GDOP within 1 to 4. The number of satellites available at each epoch should be 6.

The coordinates and receiver clock errors of the BRUX station are estimated using the ionosphere error-free combination mentioned in Table 2 and the precise satellite state vectors and satellite clock errors. The algorithm used to estimate the station's position is presented in section 3 using Ordinary Least Square (OLS) and Normal Equation Stacking (NES) methods. The ionosphere error-free measurements, number of satellites, and geometry all are the same for both methods. The estimated position is compared with the precise station position for both methods. Results are shown in Table 3. From the table, it is clear that the maximum difference between the position estimates for the NES method for dual-frequency combinations (E1, E5ab), (E1, E5a), and (E1, E5b) are 80.28cm, 96.61cm, and 82.91cm respectively. Whereas triple frequency combinations (E1, E5b, E5a), (E1, E5ab, E5a), and (E1, E5b, E5ab) are 69.28cm, 70.05cm, and 68.85cm.

Table 3
Position error (Estimated Vs Precise) for BRUX station for all dual and triple-frequency combinations for Doy 283

f1	f2	f3	OLS mean (m)	OLS 3*std (m)	NES mean (m)	NES 3*std (m)	Noise of IF (m)
E1	E5ab		1.2073	1.8792	0.552	0.2508	0.6838
E1	E5a		1.2978	2.1824	0.6502	0.3159	0.6856
E1	E5b		1.3277	1.9288	0.5576	0.2715	0.7529
E1	E5b	E5a	1.17	1.8519	0.5137	0.1791	0.6559
E1	E5ab	E5a	1.1759	1.9449	0.5209	0.1796	0.629
E1	E5b	E5ab	1.1479	1.8063	0.518	0.1705	0.6116

respectively. So, (E1, E5ab) is best for dual-frequency combinations whereas (E1, E5b, E5ab) for triple-frequency combinations is best among all frequency combinations. So, it can be stated that the position estimates improved with the use of the triple-frequency combination and NES method.

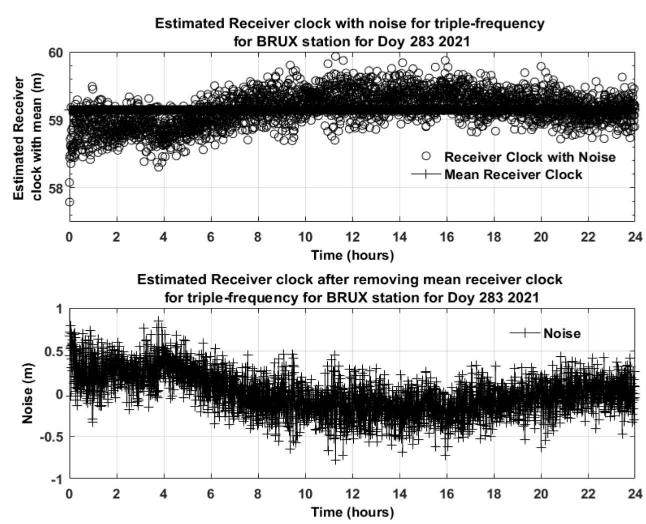

Fig. 1. Estimated Receiver clock with noise for triple-frequency combinations for BRUX station for Doy 283, 2021

The noise is generated using the receiver clock estimates for dual and triple frequency combinations. Results are shown in Fig. 1. The mean and standard deviation of the receiver clock estimates is found. The mean gives the receiver clock bias and standard deviation represents the noise of the ionosphere error-free code measurements. Alternatively, equivalent noise is obtained by removing the mean of the receiver clock as shown in Fig. 1. The total noise of an error-free code measurement combination can be obtained by multiplying the noise factor by the standard deviation. The total noise computed is tabulated in the last column of Table 3. It shows that the noise is less in triple-frequency combination than in dual-frequency. The difference between the estimated position values and the noise is smaller in the triple frequency combination.

5. Conclusions

A methodology is presented to estimate the position and receiver clock bias of the GNSS ground station using ionosphere error-free code measurements with precise satellite state vectors and satellite clock errors. Two algorithmic steps are followed to estimate the ground station's position. The algorithm for calculating the satellite's clock errors and state vectors at the time of transmission is first presented. The second algorithm is presented to find the coordinates of the ground station. It uses the iterative least square method and the MAD filter to find the coordinates and receiver clock error. These estimates are then used as an initial estimate for all other epochs. The values of the coordinates and the bias of the subsequent epoch are derived from the estimates of the Normal Equation Stacking Method.

The estimation of station position is performed for BRUX GNSS station for DOY 283, 2021 for the Galileo navigation system because of the presence of multiple frequencies measurements. The positions were estimated using the ordinary least square method and the normal equation stacking method. Also, various dual-frequency and triple-frequency combinations are analyzed to see how they affect the position estimates. The results of the least square method show the best accuracy of 295 cm whereas 68.85cm for the normal equation stacking method. The normal equation stacking method has been found to give better results than the least square method. It has also been found that (E1, E5ab) is best among dual-frequency combinations with an accuracy of 80.28cm and (E1, E5b, E5ab) best among all frequency combinations with an accuracy of 68.85cm Results indicate that the only noise remains after the estimation in triple frequency combination. It has been concluded that the station's position solution improves due to the triple frequency combination. The present method can also be used to estimate coordinates for multiple navigation systems.

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