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Estimation of Solar Radiation using Modified Lagrangian Box Model (MLBM)

Aloysius Ikechi Ugwu*
Department of Physics, FCT College of Education, Zuba, Abuja, Nigeria

***Abstract*: This study is in validating a novel solar radiation predictor model; namely, Modified Lagrangian Box Model (MLBM) by comparing its performance with global horizontal irradiance values obtained from Campbell Meteorological Station at University of Abuja. The model seeks to reduce computational complexities associated with solar radiation model which pose difficulties for ordinary household user. The MLBM estimates solar irradiance by modifying the extraterrestrial radiation normal beam with solar altitude angles. A statistical analysis comprising coefficient of residual mass (CRM), root mean squared error (RMSE), coefficient of determination (R^2) and relative percentage error (e) were used in validating the model. These performance indicators show that the model has over 80 % accuracy in predicting global horizontal irradiance (GHI) of a landscape at a local scale level.**

***Keywords*: Global horizontal, Lagrangian Box Model, Irradiance, Solar altitude angle.**

1. Introduction

Normal or direct beam, diffuse (or global horizontal irradiance, GHI) and reflected beam make up the solar resources which are assessed by widely known devices [1]. Some of these devices used in solar radiation measurement include; pyranometer, pyrheliometer and pyranometer. In solar resources, global horizontal irradiance (GHI) is the mostly sought for.

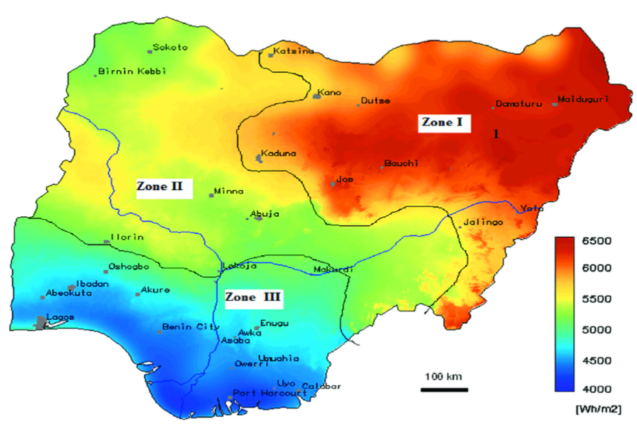

Fig. 1. Solar radiation zones for Nigeria [5]

The term irradiance used here is consider as the amount of solar energy falling on unit area over a stated time interval ($kWh.m^{-2}$) [2]. Their applications are dominantly in areas of electricity generation, thermal devices and agricultural processes. In electricity generation, solar photovoltaic cells convert solar energy directly into power supplies to homes, small and medium industries. In thermal devices, solar radiation is used in solar ovens, cookers, furnaces, heating, greenhouse and so on [3]. Nigeria is located in the tropics with high values of global solar irradiance and has three radiation zones [4], as shown in figure 1. Global solar radiation does not have constant values due to stochastics nature of weather parameters and therefore demands ground meteorological stations which are neither cheap in installation nor in maintenance to measure these values [6].

Mathematical models are being developed in overcoming these difficulties which use meteorological data to estimate global solar radiation (GSR) and there are so many of them presently being used. One of the earliest models is Angstrom's model ($H/H_0 = a + b(S_0/S)$ [7] which has been developed into linear regression in estimating GSR, H, from extraterrestrial radiation values (H_N) and sunshine hours (S_0). Others include Hargreaves and Samani model [8] ($R_s = a.R_a[T_{max} - T_{min}]^{0.5}$) that uses daily average minimum and maximum air temperatures to estimate horizontal global solar radiation (GSR) values, R_S.

Also, for the past two decades now, solar radiation models that integrate geographical information systems (GIS) in generating spatial databases of GSR have been developed [3, 9]. Solar-Flux is one of such first GIS-based solar radiation models [10]. They are rapid, cost-efficient and relatively accurate in estimating GSR of large territories [11].

In some difficult terrains like mountainous regions, models that combine satellite data have being formulated in estimating global solar radiations. Example of such model is METEOSAT which combines meteorological geostationary satellites data to predict global solar radiations [12].

There are so many other models formulated based on other climatic variables such as cloud cover, percentage of specific cloud types, evaporation, humidity, number of days with dust or smoke, air temperature, precipitation, latitude, elevation and so many immeasurable environmental interactions. Thus, the stochastic nature of climatic input variables constitutes computational complexities and equally challenges the linearity

*Corresponding author: aloyugwu@gmail.com

of the models as a simple tool every householder can use [9].

Presently, researches are geared towards finding suitable function(s) that can combine stochastic weather parameters and immeasurable environmental interactions into few parameters in the quest to reduce the computational complexities and make the models accessible for ordinary householder. Hofierka and Suri [13], following this quest, introduced the concept of energies resulting from interactions of solar radiations and atmospheric dynamics. They reiterated that the latitudinal gradients of GSR at global scale is modified by the geometry of the earth and its rotation and revolution about the sun; but made effort to show that at local scales, terrain (relief) is the major factor that creates strong local gradients in distribution of GSR.

The purpose of this study is to present capabilities, structure and performance of the MLBM in estimating global horizontal irradiance (GHI) with fewer meteorological data at local scale level, for the benefit of ordinary household user. Among the three components of solar resources (beam, diffuse and reflected), MLBM is used in estimating the diffuse component of solar resources, otherwise known as global horizontal irradiance (GHI). It can be extended to other components and at global scale level in further studies. The applications of the findings are targeted in electricity generation via photovoltaic cells and in thermal heating devices.

In contributing to this discussion, the present study focuses on formulating a novel model, MLBM that modifies the direct normal or extraterrestrial beam by using solar altitude angles at local scale level which is considered as a passive closed box. This can be used in estimating hourly, daily and monthly GHI of a landscape.

2. Modified Laggrangian Box Model (MLBM)

Lagrange equation provides the privilege of estimating the state of stationary action at any landscape which is considered as Lagragian-box (or box in short form), by the description of the difference between its dynamic and static energy functions [14]. It is based on previous works done by Angstrom [7], Hargreaves and Samani [8] and Hofierka and Suri [13]. Its feature is to reduce computational complexity inherent in present day models that discourages easy accessibility by ordinary household users.

In Page [15] work, the concept of interaction of solar radiations with the earth's surface is determined by three group-factors; namely, earth's geometry, terrain and atmospheric attenuations. His work shows that the first and the third group factors are modified by the optical thickness and air mass within the terrain. He also showed that Rayleigh criterion has provided a simple relation in determining the relative optical air mass m and optical thickness δR of the attenuating cloud within the local scale or box. Page's [15] work, he showed that this third group-factors almost determines the GHI values in the box.

Here, the state of energy in the box is the algebraic sum of interactions, between the three components of solar resources; the beam H_N, the diffuse H_{HS} and the reflected H_{RF} [16]. The beam outside the atmosphere is easier to estimate unlike the diffuse beam which is affected by climate and regional terrain. The reflected beam has little importance in affecting the values of GHI of a region. The normal beam H_N from which diffuse and reflected components are budgeted from is given by [13]:

$$H_N = S_C D_F \, exp\{-0.8662 \, T_{LK} \, m \, \delta R(m)\} \quad (1)$$

S_c is the solar constant ($1.367 \, kWm^{-2}$). D_F is eccentricity correction factor of the earth's orbit given by:

$$D_F = 1.0 + 0.033. \cos\left(2\pi(JulianDay/365)\right) \quad (2)$$

The term $-0.8662 \, T_{LK}$ is the corrected Linke atmospheric turbidity factor for air mass equal to 2 which indicates a clear sky [17]. m is the relative optical air mass calculated using the formula [18]:

$$m = (p/p_0)/\sin h_0 + 0.50572(h_0 + 6.07995)^{-1.6364} \quad (3)$$

$\delta R(m)$ is the Rayleigh optical thickness at air mass m and is calculated using improved formula by [16] as follows:

for $m \; <= \; 20$:

$$\delta R(m) = 1/(6.6296 + 1.7513 \, m - 0.1202 \, m^2 + 0.0065 \, m^3 - 0.00013 \, m^4) \quad (4)$$

for m > 20

$$\delta R(m) = 1/(10.4 + 0.718 \, m) \quad (5)$$

Global horizontal irradiance (GHI) H_{hs} ($kWhm^{-2}$) from MLBM at local surface becomes

$$H_{hs} = H_N \sin h_0 \quad (6)$$

h_0 is the solar altitude defined as angle between the sun path and the horizontal surface and it is given by [13]:

$$\sin h_0 = C31 \cos T_s + C33 \quad (7)$$

$$C31 = cos \, \phi \, cos \, \delta, \;\; C33 = sin \, \phi \, sin \, \delta \quad (8)$$

ϕ is the latitude of the location and is calculated as:

$$\phi = latitude\pi/180$$

δ is solar declination and it is expressed as:

$$\delta = (23.45\pi/180). \sin\left(284 + JulianDay/365)\right)$$

T_s measured in radians is the hour angle, calculated from the local solar time t expressed in decimal hours on the 24-hour clock as [19]:

$$T = 0.261799 \, (t - 12) \quad (9)$$

3. Research Methodology

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The samples used in this study were obtained from Campbell Scientific Automatic Weather Station, University of Abuja which is located in radiation zone II (see fig. 1); between latitude 8 °58' and longitude 7 °10' east of the Greenwich meridian [20]. Daily and monthly minimum and maximum air temperature of 2018 were used in estimating horizontal global solar radiation of Giri – Zuba District in which the Campbell Weather station is installed. The Hargreaves model stood as control to the MLBM and further validated by the GSR values recorded during the year under consideration.

Predicted horizontal global solar radiation values by Modified Lagrangian Box Model (MLBM) were compared with those obtained from Campbell Scientific Automatic Weather Station.

4. Statistical Analysis

Daily estimate of global horizontal irradiance (GHI) values from Modified Lagrangian Box Model (MLBM), (H_{HS}) were compared with measured (H_{SM}) values from Campbell Scientific Automatic Weather Station, University of Abuja. The predictive accuracy of MLBM in estimating of GHI, H_{HS} was determined using four performance indicators. The first indicator considered was the dispersion of predicted values from the line of best fit. Coefficient of Residual Mass (*CRM*), given by:

$$CRM = \frac{\sum_{i=1}^{n} H_{SM} - \sum_{i=1}^{n} H_{HS}}{\sum_{i=1}^{n} \bar{H}_{SM}} \tag{10}$$

was used in determining this. Thus, if $CRM > 0$, it indicates the tendency of over estimation, but would indicate under estimation if $CRM < 0$. Accurate prediction would occur if $CMR = 0$. $\bar{H}_{SM}$ is the mean measured GHI.

The second factor in validating MLBM is the root mean square error (*RMSE*) which is expressed as percentage and low values of it indicate accurate GHI values. RMSE is given by:

$$RMSE(\%) = \frac{\sum_{i=1}^{n} H_{HS} - \sum_{i=1}^{n} H_{SM}}{\sum_{i=1}^{n} \bar{H}_{HS}} \tag{11}$$

The third validation, Nash–Sutcliffe equation (*NSE*), also known as coefficient of determination (R^2) which measures closeness of estimated values to the best line of best fit and a model is said to performs well when its value tends to unity.

$$NSE = 1 - \frac{\sum_{i=1}^{n} (H_{SM} - H_{HS})^2}{\sum_{i=1}^{n} (H_{SM} - \bar{H}_{SM})^2} \tag{12}$$

The fourth validation factor is the relative percentage error between the mean and estimated values of GHI. It is given by:

$$e(\%) = \frac{H_{SM} - H_{HS}}{\bar{H}_{SM}} \tag{13}$$

Values of es' between -10% and +10% are considered to indicate good estimates of GHI.

5. Results and Discussions

This study attempts to contribute to efforts being made in reducing computational complexities that surround models used in estimating global horizontal irradiance, in order to reduce the difficulties, they pose to ordinary household user.

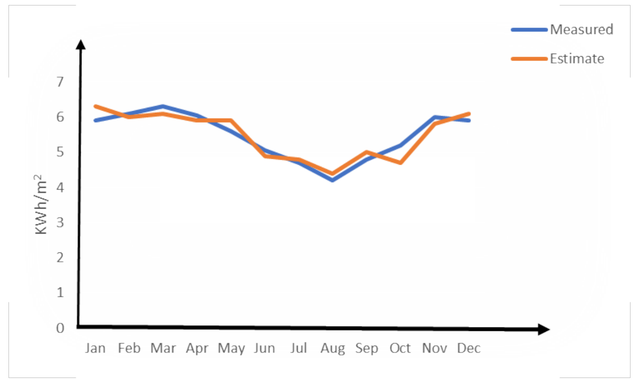

Fig. 2. Comparison of measured and estimated values of GHI

The application of MLBM to estimate daily global horizontal irradiance of Abuja using the solar altitude angle data of 2018 from Campbell weather station produced the result shown in figure 1. The result is compared with the measured GHI, H_{SM} values from the same station and equally presented on the same figure 2.

Table 1
Statistical validation of MLBM estimates against measured global horizontal irradiation values from Campbell weather station at University of Abuja

Models	CRM	RMSE	R^2	E (%)
Measured vs. MLBM Estimated Values	0.11	10.24	0.80	8.12

The performance indicator of the model is statistically analyzed and the results are presented in table 1.

The Coefficient of Residual Mass (*CRM*) shows a positive value of 0.11. This indicates the tendency of the MLBM to under-estimate the ambient solar radiation values. However, the Root Mean Squared Error (RMSE) value of 10.21%, shows that the percentage of under estimated error is small. The coefficient of determination (R^2) has a 82.00 % of the estimated GHI values agreeing with the measured values. This is further confirmed as the relative percentage error e (%) of the estimated values is 8.12 % which lies within the acceptable values; namely, - 10 % to + 10 %. The statistical analyses confirms the suitability of MLBM is estimating global horizontal irradiance at a local scale level with over 80 % accuracy.

The value of R^2 is 0.80, closed to 1 indicates that about 80.00% of the total variation is explained in the data and the value of e is small, about 12.98%. It indicates that the model estimation is good.

6. Conclusion

This study formulates a novel technique, Modified Lagragian Box Model (MLBM) in estimating Global Horizontal Irradiance at a local scale level, considered as a box of non-reactive meteorological and terrain components. It is built on Lagrangian concept of finding an action through difference between potential and kinetic energies of entities in an enclosed environment.

This is achieved through the interactions of three components of solar resources, namely; the normal beam, the diffuse beam and the reflected beam. The computation of normal beam or extraterrestrial radiation is easier to obtain than the diffuse or GHI which is highly dependent on climatic and terrestrial factors. The reflected beam has little importance in GHI radiation budgeting.

MLBM got its feeds from solar altitude angle which is used in modifying the direct beam passing through the atmosphere to estimate hourly, daily and monthly radiations of a local scale. Statistical evaluation of MLBM by comparing its performance with measured global horizontal irradiance from Campbell station of University of Abuja shows more than 80 % efficiency in predicting GHI values of local scales.

Acknowledgements

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