

Hybrid Adaptive Weighted Sliding Mode Control for Sway Suppression of a Gantry Crane System

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Abstract: This paper presents a new adaptive weighted sliding mode scheme with input shaper for sway suppression of a gantry crane system. The presented control approach combines an adaptive algorithm to a weighted-sliding controller with an input shaping technique. Input shaping reduces the transient sway and weighted sliding mode controller has multiple sliding surfaces of nonlinear gantry crane system using a single actuator. Adaptive algorithm is applied to estimate the upper bound of uncertainties and external disturbance. The stability of the closed loop system is analyzed using Lyapunov stability theory. The performances of the new hybrid control scheme are proved in MATLAB SIMULINK. Simulation results show that the presented control scheme can be used for sway suppression and precise cart positioning in the presence of uncertainties.

Keywords: Adaptive law, gantry crane, hybrid control, input shaping, residual vibration, weighted sliding mode.

1. Introduction

The main control object of a gantry crane is moving the load as fast as possible without residual sway at the desired position. However, most of the common gantry crane results in a sway motion when cart is decelerated and accelerated [5]. The swing motion can increase the moving time, even might cause accidents. The control requirement of cart positioning is that transient and residual sway of the system should be near zero or zero. A gantry crane system is the nonlinear, underactuated system and influenced by parameter variations, external disturbance and unmodelled dynamics.

The research for positioning and anti-swing of gantry cranes can be divided into two main categories. One is feedforward control technique and the other is feedback control technique. Most of the feedforward control techniques for anti-sway are input-shaping control techniques. The feedforward control technique generates the control input considering the physical and swing characteristics of the system. The feedforward control technique is sensitive to the variation of system parameters. The feedback control technique can control the pendulum by measuring the state of the system in real time, the feedback controller is robust against model uncertainties [2], [4], [7], [8], [12], [14]. Various attempts have been made to combine feedforward control technique with feedback control

technique to control gantry crane systems [3], [5], [6], [11]. Hybrid control scheme combining input shaping technique with cascade sliding mode control is proposed for a gantry crane system [6]. M.A. Ahmad, et al. [3] present a PD-type fuzzy logic control with input shaping for input tracking and anti-sway of a gantry crane system. Ahmad, et al. [5] presented LQR controller combined with modified specified negative amplitude input shaper. Dinh Do Van [7] proposed neural-fuzzy adaptive combined with an LQR controller (ANFIS-LQR). The neural-fuzzy adaptive controller can learn and control adaptively when the system parameters change. Ansu Man Singh, et al. [13] presented a control method, based on the Fast Terminal Sliding Mode (FTSM), to guarantee finite-time stability of the crane. Yang Jung Hua, et al. [14] presented adaptive nonlinear control scheme for controlling of an overhead crane to account for the parameter uncertainty.

Many studies presented effective control schemes for precise positioning and sway suppression of a nonlinear gantry crane system. However, most studies have assumed that the upper bound of model uncertainty is known or invariant. Thus, for systems with variable upper bounds of uncertainty, transient response is bad, or even chattering phenomenon is occurred. A gantry crane system is underactuated system, which should be simultaneously control the cart position and swing angle using a single actuator. In most studies on sliding mode control of a gantry crane system, there have been little work on the convergence of multiple sliding surfaces. The main challenge in underactuated SMC designs is to address the potential problems that result from interactions between the separate surfaces through control. In order to address these challenges, Layne Clemen, et al. [1] proposed designing a two-part control law that consists of a weighted summation of the single surface controls.

This paper presents a new adaptive weighted sliding mode controller with input shaper for sway suppression and precise positioning of a gantry crane system. In this work, the adaptive algorithm estimates the upper bound of uncertainties, the feed forward control scheme based on input shaping with positive Zero-vibration-derivative-derivative (ZVDD) input shaper reduces the transient sway.

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In simulations considering model uncertainties, the proposed method has been shown to reduce the swing angle to less than 0.1rad/s and the cart position error to less than 0.03m.

That is, this control approach will enable positioning and anti-swing control at the same time lifting the payload in the presence of model uncertainty.

This paper is organized as follows; Section 2 presents the mathematical model of a gantry crane system. Section 3 presents the design of hybrid adaptive weighted sliding mode control and stability analysis Section 4 demonstrate the capabilities of the control.

2. Mathematical Model of a Gantry Crane

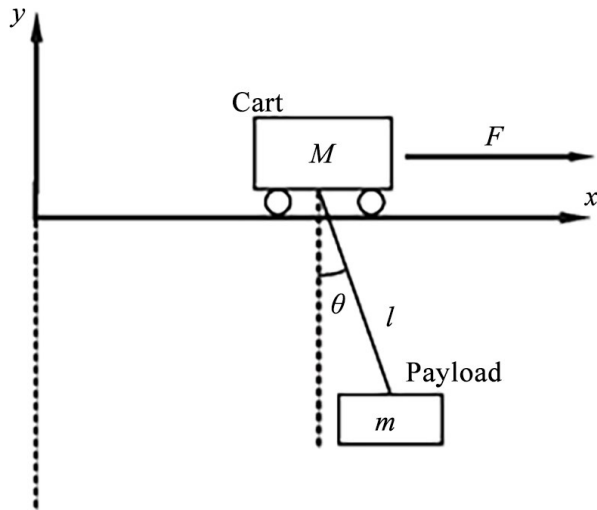


Fig. 1. Schematic diagram of a gantry crane system

Fig. 1 shows the schematic diagram of a gantry crane system.

The trolley driven by force F and payload is suspended from the trolley by a rigid rope. For simplicity, the friction force can be neglected.

The mathematics model of a gantry crane is derived by using Lagrange's equation [5], [9].

The Euler-Lagrange formulation is given as:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = F_i \quad (1)$$

where L is the Lagrangian, $\bar{q} = \{x, \theta\}$ is the generalized displacements, $\bar{F} = \{F_x, 0\}$ is the generalized forces.

Lagrangian L is formulated as,

$$L = K - U \quad (2)$$

The Kinematic energy of the system can be formulated as,

$$K = \frac{1}{2} M \dot{x}^2 + \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m l^2 \dot{\theta}^2 + m \dot{x} l^2 \sin \theta + m \dot{x} l \dot{\theta} \cos \theta \quad (3)$$

where x is the trolley position, l is the length of the rope, θ is the swing angle of payload.

The potential energy of the beam can be formulated as.

$$U = -mgl \cos \theta \quad (4)$$

The dynamic equation of a gantry crane is obtained as:

$$\ddot{x}(m+M) + ml(\ddot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta) + 2m\dot{l}\dot{\theta} \cos \theta + m\ddot{l} \sin \theta = F_x \quad (5)$$

$$l\ddot{\theta} + 2\dot{l}\dot{\theta} + \ddot{x} \cos \theta + g \sin \theta = 0 \quad (6)$$

We assume that the length changes of rope and the swing angle of payload is very small. That is $\sin \theta \approx \theta$, $\cos \theta \approx 1$, $\dot{\theta} \approx 0$, $\ddot{l} \approx 0$.

Then equation (5) and (6) are rewritten as.

$$\ddot{x}(m+M) + ml\ddot{\theta} = F_x \quad (7)$$

$$l\ddot{\theta} + \ddot{x} = 0 \quad (8)$$

Supposing that $X = [x_1 \ x_2 \ x_3 \ x_4]^T = [x \ v \ \theta \ \omega]^T$ is the state variable, the equation (7), (8) with uncertainties can be rewritten as,

$$\begin{aligned} \dot{x} &= v \\ \dot{v} &= \frac{mg}{M} \theta + \frac{1}{M} u + w_v(t) \\ \dot{\theta} &= \omega \\ \dot{\omega} &= -\frac{(M+m)g}{Ml} \theta - \frac{1}{Ml} u + w_\omega(t) \end{aligned} \quad (9)$$

Where v is the cart speed, ω is the angular velocity, $u = F_x$ the term $w_v(t)$ and $w_\omega(t)$ represent unknown disturbances.

The outputs that are to be controlled are,

$$\begin{aligned} y_1 &= x \\ y_2 &= \theta \end{aligned} \quad (10)$$

3. Design of Hybrid Adaptive Weighted Sliding Mode Controller (HAWSMC)

In this paper, a new hybrid adaptive weighted sliding mode control scheme is presented.

The presented controller scheme can improve the robustness and eliminate the transient and residual sway. HAWSMC is combined adaptive weighted sliding mode controller with input shaper. Input shaping technique is used to reduce the residual

and transient vibration. The weighted sliding mode control for sway suppression and accuracy positioning of a gantry crane system requires the upper bound of lumped uncertainty. However, length change of a rope is difficult to measure for practical application. The information of the external load disturbance is also unknown. Therefore, an adaptive law is implemented to estimate the upper bound of lumped uncertainty. The weighted sliding mode control presented in [1] can be used to control multiple sliding surface of nonlinear systems using a single actuator. The diagram of hybrid adaptive weighted sliding mode control system structure is shown in Fig. 2.

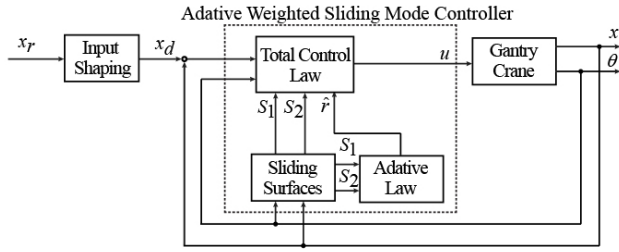


Fig. 2. Diagram of HAWSMC system

A. Input shaping technique

Input shaping technique is a feed-forward control technique.

An input shaper is a sequence of impulses. A sequence of impulses is convoluted with the desired signal. The shaped command is used to drive the system. Fig 3 shows this process. This Input shaper is composed of two impulses and will be convoluted with the unshaped input [8]. The shaped command reduces the vibration as the original set impulses.

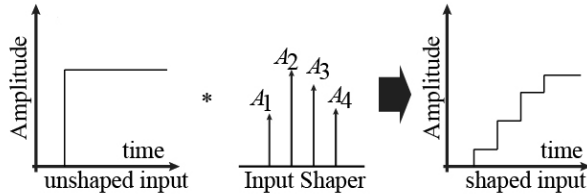


Fig. 3. Input shaping process

The design objectives of Input shaper are to determine the amplitude and time location of the impulses so that the shaped command reduces the undesired effect of the system. The time location of the impulses and amplitude are obtained from natural frequencies and damping ratio of the system [3].

The oscillating system can be considered as second order system.

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (11)$$

where ω_n is the natural frequency of the oscillation system, ζ is the damping ration of the system.

In this paper, the presented HAWSMC is combined with the Zero-Vibration-Derivative-Derivative (ZVDD) shaper [3].

The positive ZVDD input shaper is a four-impulse sequence,

which can be obtained with the parameters as,

$$\begin{aligned} t_1 &= 0, t_2 = \frac{\pi}{\omega_d}, t_3 = \frac{2\pi}{\omega_d}, t_4 = \frac{3\pi}{\omega_d} \\ A_1 &= \frac{1}{1+3K+3K^2+K^3}, A_2 = \frac{3K}{1+3K+3K^2+K^3} \\ A_3 &= \frac{3K^2}{1+3K+3K^2+K^3}, A_4 = \frac{K^3}{1+3K+3K^2+K^3} \end{aligned} \quad (12)$$

where,

$$K = e^{-\zeta\pi(1-\zeta)^{-1/2}}$$

$$\omega_d = \omega_n(1-\zeta)^{1/2}$$

For the impulses, t_j and A_j are the time location and amplitude of impulse respectively.

B. Adaptive Weighted Sliding mode controller

The sliding surfaces used for weighted sliding mode controller [1] are of the form,

$$S_k = \dot{e}_k + \sigma_k e_k \quad (13)$$

Where $k \in \{1, 2\}$, $e_k = y_k - y_{kd}$, y_k is the k^{th} output, y_{kd} is the k^{th} desired output. The two sliding surfaces track a desired cart position (x_d) and regulate the angle (θ) to 0. We assume that the desired cart position is a constant.

From equation (13), the two sliding surfaces are defined as,

$$\begin{aligned} S_1 &= v + \sigma_1(x - x_d) \\ S_2 &= \omega + \sigma_2\theta \end{aligned} \quad (14)$$

The auxiliary control laws can be defined as follows,

$$\begin{aligned} u_1 &= -M\left(\frac{mg}{M}\theta + \sigma_1 v\right) - M\hat{r} \operatorname{sgn}(S_1) \\ u_2 &= -Ml\left(-\frac{(M+m)}{Ml}g\theta + \sigma_2 \omega\right) + M\hat{r} \operatorname{sgn}(S_2) \end{aligned} \quad (15)$$

Where \hat{r} is the estimated value of \bar{r} , \bar{r} is an unknown but bounded positive constant.

The uncertainties in equation (9) is unknown but bounded, i.e. $|w_v| < \bar{r}$, $|\bar{w}_\omega| < \bar{r}$.

From equation (15), the total control law is given by,

$$u = \frac{\frac{1}{M}\alpha_1 S_1 u_1 + \frac{1}{Ml}\alpha_2 S_2 u_2}{\frac{1}{M}\alpha_1 S_1 + \frac{1}{Ml}\alpha_2 S_2} \quad (16)$$

where α_1, α_2 are positive constants.

In equation (15), \hat{r} is the estimated value by \bar{r} . the following adaptive algorithm for the bound of $|w_v|, |w_\omega|$ is considered as,

$$\dot{\hat{r}} = \frac{1}{\beta} (\alpha_1 |S_1| + \alpha_2 |S_2|) \quad (17)$$

where β is the adaptation gain ($\beta > 0$).

The adaptive algorithm in equation (17) was proposed to estimate the upper bound of the uncertainties.

In order to reduce the chattering phenomena, the sign function $\text{sgn}(S_1)$ can be replaced by a hyperbolic tangent function $g(s) = (1 - e^{-\mu s}) / (1 + e^{-\mu s})$, ($\mu > 1$).

C. Stability analysis of AWSMC

The time derivative of S_k is given by,

$$\begin{aligned} \dot{S}_1 &= \dot{v} + \sigma_1 v = \frac{mg}{M} \theta + \frac{1}{M} u + w_v(t) + \sigma_1 v = \\ &= \Omega_1(X, u) + w_1(t) \\ \dot{S}_2 &= \dot{\omega} + \sigma_2 \omega = \\ &= -\frac{(M+m)g}{ML} \theta - \frac{1}{ML} u + w_\omega(t) + \sigma_2 v = \\ &= \Omega_2(X, u) + w_2(t) \end{aligned} \quad (18)$$

where Ω_i is an implicit function of S_i , $w_1(t) = w_v(t)$, $w_2(t) = w_\omega(t)$.

The Lyapunov function candidate is defined as,

$$V = \sum_{i=1}^2 \alpha_i V_i + \frac{1}{2} \beta \tilde{r}^2 \quad (19)$$

$$V_k = \frac{1}{2} S_k^2, \quad k=1,2 \quad (20)$$

where $\tilde{r} = \hat{r} - \bar{r}$.

Then, the time derivative of V is given by,

$$\begin{aligned} \dot{V} &= \sum_{k=1}^2 \alpha_k \dot{V}_k(S_k) \dot{S}_k + \beta \tilde{r} \dot{\hat{r}} \\ &= \sum_{k=1}^2 \alpha_k \dot{V}_k(S_k) [\Omega_k(X, u) + w_k(t)] + \beta \tilde{r} \dot{\hat{r}} \\ &= \sum_{k=1}^2 \alpha_k \dot{V}_k(S_k) [\Omega_k(X, u_k) + w_k(t)] + \\ &+ \sum_{k=1}^2 \alpha_k \dot{V}_k(S_k) [\Omega_k(X, u) - \Omega_k(X, u_k)] + \beta \tilde{r} \dot{\hat{r}} \end{aligned} \quad (21)$$

Where,

$$\begin{aligned} \alpha_1 \dot{V}_1(S_1) [\Omega_1(X, u_1) + w_1(t)] &= \\ &= \alpha_1 S_1 \left[\frac{mg}{M} \theta + \frac{1}{M} u_1 + w_v(t) + \sigma_1 v \right] \\ &= \alpha_1 S_1 \left\{ \frac{mg}{M} \theta + \frac{1}{M} \left[-M \left(\frac{mg}{M} \theta + \sigma_1 v \right) - M \hat{r} \text{sgn}(S_1) \right] \right\} \end{aligned} \quad (22)$$

$$\begin{aligned} &+ \alpha_1 S_1 w_v(t) + \alpha_1 S_1 \sigma_1 v \\ &= \alpha_1 S_1 [w_v(t) - \hat{r} \text{sgn}(S_1)] \leq \alpha_1 |S_1| |w_v(t)| - \alpha_1 S_1 \hat{r} \text{sgn}(S_1) \\ &= \alpha_1 |S_1| |w_v(t)| - \alpha_1 \hat{r} |S_1| = \alpha_1 |S_1| (|w_v(t)| - \hat{r}) \end{aligned}$$

$$\begin{aligned} \alpha_2 \dot{V}_2(S_2) [\Omega_2(X, u_2) + w_2(t)] &= \\ &= \alpha_2 S_2 \left[-\frac{(M+m)g}{ML} \theta - \frac{1}{ML} u_2 + w_\omega(t) + \sigma_2 v \right] \\ &= \alpha_2 S_2 [w_\omega(t) - \hat{r} \text{sgn}(S_2)] \\ &\leq \alpha_2 |S_2| |w_\omega(t)| - \alpha_2 S_2 \hat{r} \text{sgn}(S_2) \\ &= \alpha_2 |S_2| |w_\omega(t)| - \alpha_2 |S_2| \hat{r} = \alpha_2 |S_2| (|w_\omega(t)| - \hat{r}) \end{aligned} \quad (23)$$

or

$$\begin{aligned} \sum_{k=1}^2 \alpha_k \dot{V}_k(S_k) [\Omega_k(X, u) - \Omega_k(X, u_k)] &= \\ &= \alpha_1 S_1 \frac{1}{M} (u - u_1) + \alpha_2 S_2 \frac{1}{ML} (u - u_2) \\ &= \left(\alpha_1 S_1 \frac{1}{M} + \alpha_2 S_2 \frac{1}{ML} \right) u - \alpha_1 S_1 \frac{1}{M} u_1 - \alpha_2 S_2 \frac{1}{ML} u_2 \\ &= 0 \end{aligned} \quad (24)$$

From equation (22), (23) and (24), equation (21) can be rewritten as,

$$\begin{aligned} \dot{V} &\leq \alpha_1 |S_1| (|w_v(t)| - \hat{r}) + \alpha_2 |S_2| (|w_\omega(t)| \\ &\quad - \hat{r}) + \beta \tilde{r} \dot{\hat{r}} \\ &= \alpha_1 |S_1| (|w_v(t)| - \hat{r}) + \alpha_2 |S_2| (|w_\omega(t)| - \hat{r}) \\ &\quad + \beta (\hat{r} - \bar{r}) \dot{\hat{r}} \end{aligned} \quad (25)$$

$$= \alpha_1 |S_1| (|w_v(t)| - \hat{r}) + \alpha_2 |S_2| (|w_\omega(t)| - \hat{r}) + (\hat{r} - \bar{r}) (\alpha_1 |S_1| + \alpha_2 |S_2|)$$

$$= \alpha_1 |S_1| (|w_v(t)| - \bar{r}) + \alpha_2 |S_2| (|w_\omega(t)| - \bar{r})$$

Then

$$\dot{V} < 0 \quad (26)$$

By the Lyapunov stability theory, the not only two sliding surfaces converge to zero, but also the estimated upper bound of uncertainty converges to real upper bound of uncertainty.

4. Simulation

In this section, the simulation results of a hybrid adaptive weighted sliding mode controller for precise positioning and sway suppression of gantry crane system are presented. To compare the performance of the proposed controller, the simulation of the fuzzy controller with Input shaper, PD controller are also performed.

The gantry crane parameter are as follows,

$$x_r = 20m, \quad x(0) = 0m, \quad v(0) = 0m/s, \quad M = 20000kg, \\ m = 1000kg, \quad l = 6m, \quad g = 9.81m/s^2$$

The system with the disturbances $w_v = \sin 10t$, $w_\theta = \cos 5t$.

The parameters of input shaping are obtained as: $A_1 = 0.2811$, $A_2 = 0.4440$, $A_3 = 0.2338$, $A_4 = 0.0410$, $T = 4.9163$. the parameters of adaptive weighted sliding mode controller are selected as: $\alpha_1 = 1$, $\alpha_2 = 1$, $\sigma_1 = 0.2$, $\sigma_2 = 1.5$, $\beta = 10$.

According to the hybrid adaptive weighted sliding mode control scheme, the simulation results with disturbance are shown in Fig. 4, 5. Fig. 4 is the cart position curve. Fig 5 is the curve of swing angle.

From the simulation results, it has proved that the HAWSMC can realize the precise position control of the cart, the transient and residual sway of payload can be reduced significantly.

Compared with the generic SMC and PD-type fuzzy controller with input shaping, the proposed control scheme can control precise cart position, and reduce the transient and residual vibration more efficiently. the simulation results show that the proposed scheme can eliminate chattering phenomenon.

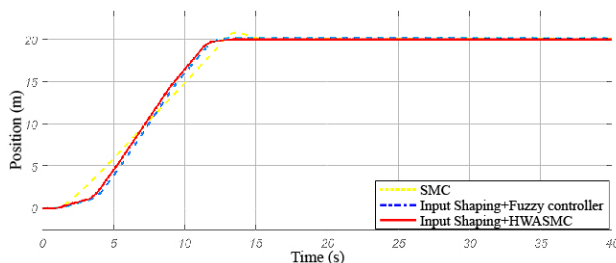


Fig. 4. Cart position curve with disturbance

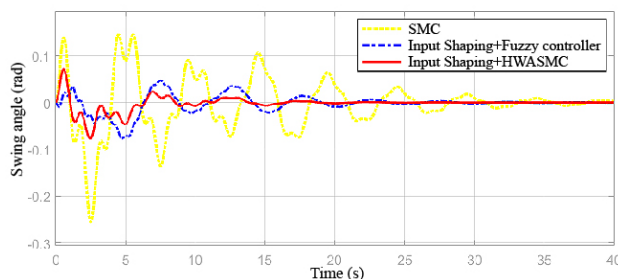


Fig. 5. The curve of swing angle with disturbance

5. Conclusion

In this paper, a new hybrid adaptive weighted sliding mode control scheme is proposed for the underactuated gantry crane system. the adaptive weighted sliding mode control which combines input shaping technique is proposed to implement the cart position and reduce the transient sway. the positive ZVDD input shaper reduced the transient vibration and the adaptive weighted sliding mode controller realized simultaneous the residual sway suppression and precise cart positioning. the chattering phenomenon has been also eliminated. the simulation result proved its effectiveness.

1. We conclude by giving some directions for future works. Considering the saturation prevention of adaptive law in real computations.
2. Considering the selection of the gains of multiple sliding surfaces.
3. Considering the selection of adaptation gain in adaptive algorithm.

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