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Segmented Regression and its Applications: Structural Break Analysis of Retail Gold Prices in Kerala

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Abstract: **Segmented regression, also known as piecewise regression or broken-stick regression, is an important statistical technique used to model relationships where the underlying mechanism changes at one or more unknown thresholds or breakpoints. This research article provides a comprehensive theoretical review of segmented regression models, covering continuous and discontinuous specifications for both single and multiple breakpoints, parameter estimation via Ordinary Least Squares (OLS) and Muggeo's iterative linearization algorithm, and inferential procedures including hypothesis testing for structural breaks. We then apply this methodology to a real-world financial time series of daily retail gold prices (per 8 grams) in the Kerala market from April 2025 to April 2026 (n = 395 observations). Using Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) for model selection, we identify three statistically robust structural breakpoints occurring on August 4, 2025, January 13, 2026, and January 29, 2026. These breakpoints divide the price series into four distinct pricing regimes characterized by accelerating upward drift followed by a sharp correction. Diagnostic evaluations including residual analysis, Shapiro-Wilk normality testing, and Durbin-Watson autocorrelation testing are conducted to validate the model and address methodological limitations.**

Keywords: **Segmented Regression, Piecewise Linear Models, Structural Breakpoints, Muggeo Algorithm, Gold Price Dynamics, Interrupted Time Series.**

1. Introduction

Segmented regression, widely referred to as piecewise regression, broken-stick regression, or change-point regression, is a powerful statistical technique designed to model complex relationships where the functional form of the regression equation shifts at one or more specific points along the domain of the independent variable [19]. Unlike standard global linear or nonlinear regression models which assume a uniform relationship across the entire dataset, segmented regression partitions the data domain into contiguous segments, fitting separate regression equations within each segment [24].

The methodology finds extensive applications across diverse scientific and economic domains:

- Epidemiology and Public Health: Assessing the immediate and long-term impact of public health interventions and policy changes through interrupted time series (ITS) analysis [17], [26].
- Economics and Finance: Identifying structural breaks, regime shifts, and threshold effects in macroeconomic growth rates, inflation dynamics, and commodity prices [3], [23].
- Environmental Science: Capturing ecological response thresholds where environmental pollutants begin to exert measurable harmful effects [18].
- Engineering: Modelling physical stress-strain relationships exhibiting distinct linear regions before and after yield points.

The central methodological challenge in segmented regression lies in the fact that breakpoint locations (changepoints or knots) are typically unknown a priori and must be estimated directly from the data alongside segment-specific slope and intercept parameters [11]. While grid search methods and profile likelihood evaluation are conceptually straightforward, they can be computationally intensive for multiple breakpoints [13]. More advanced algorithmic frameworks, such as Muggeo's iterative linearization procedure, facilitate efficient parameter estimation in generalized linear models [19].

The objectives of this article are fourfold: (1) to review the theoretical and mathematical framework of segmented regression models; (2) to examine estimation methodologies with a focus on Muggeo's iterative linearization algorithm; (3) to apply this framework empirically to daily retail gold prices in Kerala (April 2025–April 2026) to detect structural breakpoints; and (4) to validate the models through rigorous residual diagnostics and discuss substantive economic implications.

The rest of the paper is organized as follows. Section 2 traces the historical development of structural break testing (Quandt, Chow), asymptotic distribution theory (Hinkley, Feder), algorithmic advances (Muggeo's iterative linearization, Bai-Perron multiple breaks), and empirical applications in epidemiology, climate science, and finance. In Section 3 formulates the mathematical foundation of segmented regression, covering continuous/discontinuous single and multiple breakpoints, OLS estimation for fixed knots,

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Muggeo's iterative updating algorithm, model evaluation criteria (AIC/BIC), and residual diagnostics. Section 4 applies the methodology to daily retail gold prices in Kerala (n = 395), presenting model selection metrics, estimated structural breakpoints, regime-wise slopes, and residual diagnostic evaluations supported by embedded graphs. And finally, the summaries of major empirical findings, substantive economic implications, statistical limitations, and future research directions are presented in Section 5.

2. Review of Literature

A. Early Theoretical Foundations

The formal statistical investigation of structural breaks in regression models originated in the mid-twentieth century. Quandt (1958) [23] introduced a switching regression framework assuming observations belong probabilistically to one of two distinct regimes. Chow (1960) [6] extended this by developing a formal test for equality of regression coefficients across sub-periods, which remains a cornerstone of econometric structural break testing, conditional on known breakpoint locations.

Subsequent theoretical advancements established asymptotic properties for changepoint estimators. Hinkley (1969, 1971) [11], [12] derived maximum likelihood estimator distributions for single-phase intersections under Gaussian errors, demonstrating faster convergence rates than standard root-n asymptotic behavior due to high local information density around changepoints. Feder (1975) [8] established asymptotic normality for least-squares estimators in continuous bent-line regression. Comprehensive treatments of linear model assumptions and diagnostics by Kutner et al. (2005) [15] provide essential methodological safeguards against heteroscedasticity, non-normality, and model misspecification. Furthermore, Seber and Wild (2003) [24] emphasized piecewise modelling as an elegant middle ground between global linearity and fully nonparametric regression.

B. Algorithmic Advances and Multiple Breakpoints

Addressing unknown breakpoint locations without exhaustive grid searches led to significant computational breakthroughs. Muggeo (2003) [19] introduced an iterative linearization algorithm using first-order Taylor expansions to approximate segmented regression functions around current estimates, allowing breakpoint updates via ordinary least squares. This technique was implemented in the widely used segmented R package [20]. Concurrently, Bai and Perron (1998, 2003) [3], [4] formulated a comprehensive dynamic programming framework for estimating and testing multiple structural changes in linear models. Liu et al. (1997) [16] introduced the LWZ information criterion to penalize model complexity and determine optimal breakpoint counts.

Parallel methodological developments include Bayesian changepoint analysis via product partition models [5] and reversible-jump MCMC [10], structural change testing frameworks in econometrics [1], [28], and Wild Binary Segmentation for high-frequency or noisy change-point detection [9], [25].

C. Empirical Applications in Public Health, Climate, and Finance

Segmented regression has achieved widespread adoption across applied disciplines. In public health, Wagner et al. (2002) [26] and Lopez Bernal et al. (2017) [17] established interrupted time series as a gold standard for evaluating policy interventions, a framework recently adapted by Muggeo et al. (2020) [22] to model COVID-19 epidemic trajectories and lockdown efficacy. In climate science, Menne (2005) [18] and Werner et al. (2015) [27] applied piecewise regression to identify century-scale temperature regime shifts. In financial economics, Perron (1989) demonstrated that macroeconomic unit roots often reflect deterministic structural breaks, while recent work explores threshold effects in growth and inequality [14].

D. Identification of the Research Gap

While segmented regression is well-established in climate science, macroeconomics, and epidemiology, its application to contemporary high-volatility precious metal markets–specifically regional retail gold pricing during periods of intense geopolitical and fiscal uncertainty–remains underexplored. Conventional financial models (such as GARCH or global linear trends) often smooth out structural regime shifts. Applying Muggeo's iterative framework to daily Kerala gold retail prices (2025–2026), validated through rigorous diagnostic principles [15], addresses this empirical gap.

3. Theoretical Framework

A. The Simple Segmented Regression Model (One Breakpoint)

The standard continuous piecewise linear model with a single breakpoint ψ is formulated as:

$$Y = \beta_0 + \beta_1 X + \beta_2 (X - \varphi) + \epsilon \quad (1)$$

where$(X - \varphi)_+ = \max(0, X - \varphi)$ represents the hinge function. Here, β_0 is the intercept, β_1 is the initial slope for$X < \varphi$, and β_2 represents the change in slope at the breakpoint, such that the slope of the second segment is $\beta_1 + \beta_2$. Continuity is automatically preserved at $X = \varphi$.

An equivalent parameterization utilizes separate intercepts and slopes:

$$Y = \alpha_1 + \beta_1 X \quad if\ X < \varphi \quad (2)$$

$$Y = \alpha_2 + \beta_2 X \quad if\ X \geq \varphi \quad (3)$$

subject to the join-point constraint $\alpha_1 + \beta_1 \varphi = \alpha_2 + \beta_2 \varphi$..

B. Multiple Breakpoints

For K ordered breakpoints $\varphi_1 < \varphi_2 < \cdots < \varphi_k$ the continuous piecewise linear model extends to:

$$Y = \beta_0 + \beta_1 X + \sum_{k=1}^{K} \gamma_k (X - \varphi_k) + \epsilon \quad (4)$$

where γ_k denotes the slope increment at breakpoint φ_k.

C. Estimation Methodology: Muggeo's Iterative Linearization

When breakpoints are unknown, Muggeo (2003) [19] linearizes the term $(X - \varphi)_+$ around a current estimate $\varphi^{(r)}$ via Taylor expansion:

$$(X - \varphi)_+ \approx \left(X - \varphi^{(r)}\right)_+ - \left(\left(\varphi - \varphi^{(r)}\right)\middle|\left(X - \varphi^{(r)}\right)\right) \quad (5)$$

Defining auxiliary variables $\mathrm{U} = \left(\mathrm{X} - \varphi^{(\mathrm{r})}\right)_+$ and $\mathrm{V} = \left|\left(\mathrm{X} > \varphi^{(\mathrm{r})}\right)\right.$, the model becomes linear in augmented regressors. Ordinary least squares updates the slope change γ and breakpoint increment $\delta = \varphi - \varphi^{(\mathrm{r})}$, yielding $\varphi^{(\mathrm{r}+1)} = \varphi^{(\mathrm{r})} + \hat{\delta}$, Iteration continues until convergence.

D. Model Selection and Diagnostics

Model selection across competing breakpoint counts utilizes Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC):

$$A|C = -2\ln L\left(\hat{\beta}, \hat{\varphi}\right) + 2\rho \quad (6)$$

$$B|C = -2\ln L\left(\hat{\beta}, \hat{\varphi}\right) + \rho \ln n \quad (7)$$

where p is the number of estimated parameters and n is sample size. Residual diagnostics include Shapiro-Wilk tests for normality, Durbin-Watson statistics for autocorrelation, and Cook's distance for influential observations [15].

4. Empirical Application: Kerala Gold Prices (2025–2026)

A. Data Description

The empirical analysis examines daily retail gold prices (per 8 grams) in Kerala, sourced from keralagoldrates.com, spanning April 1, 2025 to April 30, 2026 (n = 395 consecutive daily observations). Prices increased from INR 68,080 to a peak of INR 130,360 on January 29, 2026, before settling at INR 112,000 on April 30, 2026.

B. Model Selection Results

Continuous piecewise linear models with 0 to 4 breakpoints were fitted using Muggeo's iterative algorithm. Table 1 displays model performance metrics.

While R^2 increases monotonically, AIC and BIC reach their optimal penalty balance at 3 breakpoints. The 3-breakpoint model was selected as the final specification.

Table 1
Comparison of segmented regression model by number of breakpoints

Model Specification	RSS	AIC	BIC	R^2
Linear (0 breakpoints)	1.171×10^{10}	7922.8	7934.7	0.9028
1 breakpoint	8.486×10^{9}	7797.7	7813.6	0.9296
2 breakpoint	4.064×10^{9}	7508.9	7528.8	0.9663
3 breakpoint	2.877×10^{9}	7374.5	7398.3	0.9761
4 breakpoint	2.653×10^{9}	7344.4	7372.3	0.9780

C. Estimated Breakpoints and Regime Slopes

Table 2 presents the estimated structural breakpoints and their substantive interpretations.

D. Residual Diagnostics and Graphical Evaluation

Residual standard error was established at INR 2,716. The Shapiro-Wilk test indicated departure from normality ($W = 0.94, p < 0.001$), primarily driven by extreme price volatility around the January 2026 peak. The Durbin-Watson statistic ≈ 0.21 revealed strong positive autocorrelation, an expected artifact of daily financial time series that suggests standard errors are understated.

To visually evaluate the model fit and diagnostic properties, Figures 1–4 illustrate the empirical results.

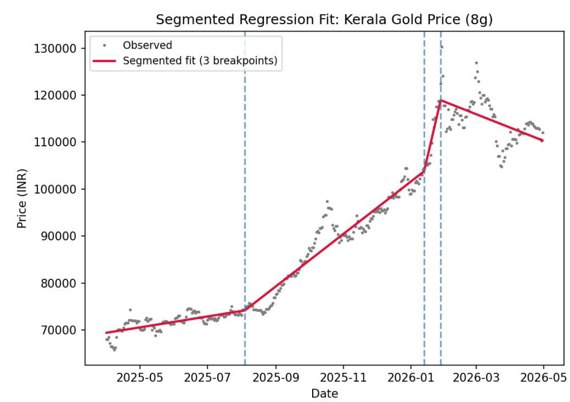

Fig. 1. Segmented regression fit for Kerala retail gold prices (n = 395) showing the three estimated structural breakpoints

Table 2
Estimated structural breakpoints in Kerala retail gold prices

Breakpoint	Estimated Date	Day Index(t)	Market Interpretation
φ_1	August 4, 2025	≈ 116	End of post-Eid correction; transition to sustained steady uptrend.
φ_2	January 13, 2026	≈ 288	Abrupt acceleration into parabolic rally phase.
φ_3	January 29, 2026	≈ 303	Peak of the rally; onset of post-peak price correction.

Table 3
The segment-wise slopes and monthly growth rates

Segment	Period	Slope (INR/day)	Approx. Monthly Change (INR)
1	01-Apr-2025 to 04-Aug-2025	38.0	~ +1, 140
2	04-Aug-2025 to 13-Jan-2026	182.8	~ +5, 480
3	13-Jan-2026 to 29-Jan-2026	1008.8	~ +30, 260
4	29-Jan-2026 to 30-Apr-2026	-92.9	~ −2, 790

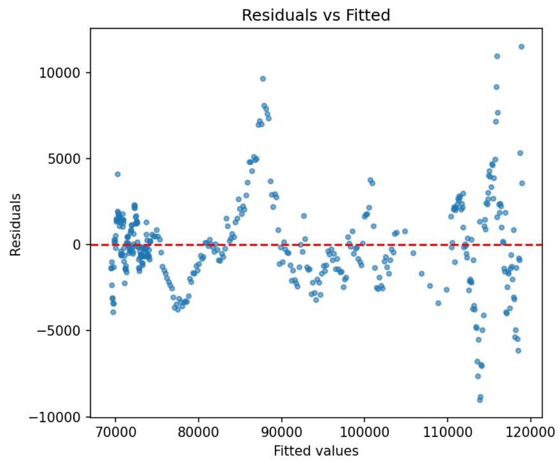

Fig. 2. Residuals versus fitted values from the three-breakpoint segmented model

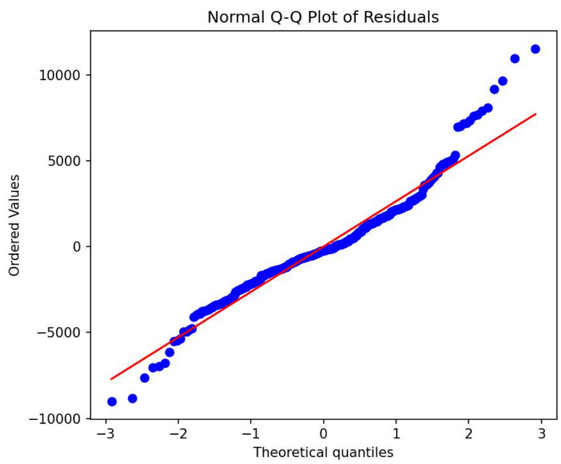

Fig. 3. Normal Q-Q plot of the residuals from the fitted segmented model

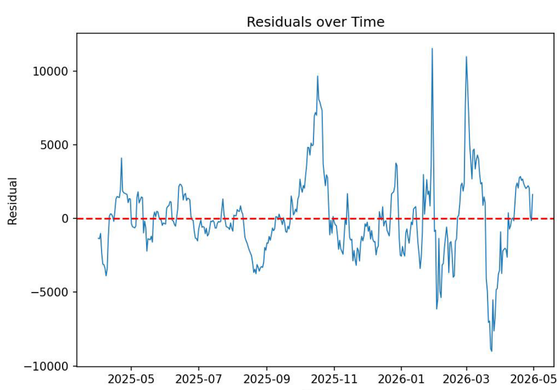

Fig. 4. Residuals plotted over time, highlighting positive autocorrelation

5. Conclusion

This study demonstrated that retail gold price dynamics in Kerala during 2025–2026 are highly non-linear and characterized by distinct structural regime shifts. Using Muggeo's iterative segmented regression framework, we identified three significant breakpoints dividing the 395-day series into four distinct phases: a steady drift, a moderate acceleration, a parabolic spike exceeding INR 1,000 per day, and a subsequent correction. While daily financial autocorrelation limits strict asymptotic precision, the 3-breakpoint model $R^2 = 0.976$ provides a robust analytical tool for market participants, analysts, and policymakers navigating commodity price volatility.

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